

THE APPLICATION OF THE HYDRAULIC ANALOGIES  
TO PROBLEMS OF TWO-DIMENSIONAL COMPRESSIBLE GAS FLOW

42  
R. Rusby  
Crosland

A THESIS

Presented to

the Faculty of the Division of Graduate Studies

Georgia School of Technology

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Aeronautical Engineering

by

John Elmer Hatch, Jr.

March 1949

THE APPLICATION OF THE HYDRAULIC ANALOGIES  
TO PROBLEMS OF TWO-DIMENSIONAL COMPRESSIBLE GAS FLOW

Approved:


Date Approved by Chairman June 2, 1949

## ACKNOWLEDGMENTS

I wish to express my sincerest appreciation to Professor H. W. S. LaVier for his valuable aid and guidance in all phases of the work required for the completion of this project. I should also like to thank Mr. R. S. Leonard, of the Engineering Experiment Station Machine Shop, and Mr. J. E. Garrett, of the Engineering Experiment Station Photographic Laboratory, for their full cooperation in the use of their respective facilities. Constructive criticism and suggestions of the staff of the Daniel Guggenheim School of Aeronautics greatly helped in the prosecution of the problems involved in this work.

## TABLE OF CONTENTS

	PAGE
Approval Sheet.....	ii
Acknowledgements.....	iii
List of Figures.....	v
Summary.....	1
Introduction.....	1
Theory.....	4
Equipment.....	13
Procedure.....	18
Discussion.....	19
Conclusions.....	23
Recommendations.....	23
BIBLIOGRAPHY.....	25
APPENDIX I, Figures.....	27



## LIST OF ILLUSTRATIONS

<u>FIGURE NO.</u>	<u>TITLE</u>	<u>PAGE NO.</u>
1.	General View of Water Channel.....	28
2.	General View of Water Channel.....	29
3.	Top View of Variable Speed Drive Mechanism.....	30
4.	Schematic Diagram of Water Channel and Lighting Equipment.....	31
5.	Shock Polar Diagrams For Diamond Airfoil Test Speeds.....	32
6.	Shock Polar Diagrams For Circular Arc Airfoil Test Speeds.....	33
7.	Shock Polar Diagrams.....	34
8.	Diamond Airfoil Model.....	35
9.	Circular Arc Airfoil Model.....	36
10.	Model Airfoil Angle of Attack Calibration Curve.....	37
11.	Variation of Wave Velocity With Wave Length For A Static Water Depth of One Quarter of an Inch.....	38
12.	Analysis of Refraction Patterns Formed By An Undular Hydraulic Jump, and Capillary Waves.....	39
13.	Flow About Circular Arc Airfoil, $M = 1.56$ .....	40
14.	Flow About Circular Arc Airfoil, $M = 2.34$ .....	41
15.	Flow About Circular Arc Airfoil, $M = 3.28$ .....	42
16.	Flow About Lenticular Airfoil, $M = 3.71$ .....	43
17.	Flow About Lenticular Airfoil, $M = 4.26$ .....	44
18.	Flow About Lenticular Airfoil, $M = 6.78$ .....	45
19.	Flow About Diamond Airfoil, $M = 2.63$ .....	46
20.	Flow About Diamond Airfoil, $M = 2.47$ .....	47

## LIST OF ILLUSTRATIONS

<u>FIGURE NO.</u>	<u>TITLE</u>	<u>PAGE NO.</u>
21.	Flow About Diamond Airfoil, $M=3.02$ .....	48
22.	Flow About Diamond Airfoil, $M=3.72$ .....	49
23.	Flow About Diamond Airfoil, $M=4.53$ .....	50
24.	Flow About Diamond Airfoil, $M=5.08$ .....	51
25.	Variation of Shock Wave Angle With Free Stream Mach Number For Diamond Airfoil.....	52
26.	Variation of Shock Wave Angle With Free Mach Number For Symetrical Circular Arc Airfoil.....	53
27.	Diamond Airfoil Local Mach Number Distribution For Various Free Stream Mach Numbers.....	54
28.	Circular Arc Airfoil Local Mach Number Distribution For Various Free Stream Mach Numbers.....	55
29.	Pressure Coefficient For Diamond Airfoil at $M_s = 4.53$ .....	56
30.	Pressure Coefficient For Circular Arc Airfoil at $M_s = 4.26$ .....	57



THE APPLICATION OF THE HYDRAULIC ANALOGIES  
TO PROBLEMS OF TWO-DIMENSIONAL COMPRESSIBLE GAS FLOW

SUMMARY

A water channel, with a glass bottom, four feet wide and twenty feet long, was designed and constructed for the purpose of investigating the application of the hydraulic analogy to problems of two-dimensional compressible flow.

Two supersonic airfoil profiles were tested at various speeds. The speed range covered by analogy was from  $M = .75$  to  $M = 6.0$ . Pictures were taken of the wave formations set up by each airfoil model. Pictures were also taken of the local water depth distribution about the models. From a study of the flow photographs, and by applying the hydraulic analogy, an analysis was made of the local conditions about the models.

INTRODUCTION

There is an analogy between water flow with a free surface, and two-dimensional compressible gas flow. The analogy may be used for the investigation of aerodynamic problems in high speed compressible gas flow. Where no supersonic or high speed wind tunnels are available, the application of the hydraulic analogy is particularly useful for the study of compressible gas flow problems.

Riabouchinsky<sup>1</sup> first presented the mathematical basis for the hydraulic analogy, and applied the analogy to the investigation of the flow in a Laval nozzle. Binnie and Hooker<sup>2</sup> further investigated the application of the hydraulic analogy by obtaining surveys along the center of a channel with a constriction. Entirely satisfactory agreement between compressible gas flow theory and the hydraulic analogy was reached. Ernst Preiswerk,<sup>3</sup> at the suggestion of Dr. J. Ackeret, conclusively proved that the methods of gas dynamics can be applied to water flow with a free surface. In 1940 the National Advisory Committee for Aeronautics constructed a water channel for the investigation of the hydraulic analogy to the flow through nozzles and about circular cylinders at subsonic velocities and extending into the supercritical velocity range.<sup>4</sup> Reasonable satisfactory agreement was found between the water flow and air flow about similar bodies, although it was concluded from this work that accurate quantitative

---

<sup>1</sup>Riabouchinsky, D., *Mecanique des fluides. Comptes Rendus*, t. 195, No. 22, November 28, 1932, pp. 998-999.

<sup>2</sup>Binnie, A.M., and S. G. Hooker, "The Flow Under Gravity of an Incompressible and Inviscid Fluid Through a Constriction in a Horizontal Channel", *Proceedings of the Royal Society*, Vol. 159 (London, England: 1937), pp. 592-608.

<sup>3</sup>Preiswerk, Ernst., "Application of the Methods of Gas Dynamics to Water Flows With Free Surface."

Part I. "Flows with No Energy Dissipation." *NACA TM*, No. 934, 1940.

Part II. "Flows with Momentum Discontinuities." *NACA TM*, No. 935, 1940.

<sup>4</sup>James Orlin, Norman J. Linder, and Jack G. Bitterly, "Application of the Analogy Between Water Flow With a Free Surface and Two-Dimensional Compressible Gas Flow." *NACA TM*, No. 1185, 1947.



results would require additional investigations, both theoretical and experimental. North American Aviation Incorporated recognized the possibilities of extending experimental compressibility research by means of the hydraulic analogy, and in 1946 constructed a water channel in which the water remained stationary and the test models moved through the water. It was indicated by North American Aviation Incorporated that with accurate equipment, quantitative as well as qualitative results could be expected from water channel experiments.<sup>5</sup>

The hydraulic analogy offers a convenient and inexpensive way of investigating two-dimensional compressible gas flow of phenomena occurring in air at speeds too high for visual observation. By means of the water channel, the two-dimensional flow in air at high Mach numbers can be simulated to low speeds, and the results may be easily photographed.

The advantages of using the hydraulic analogy for investigating problems of high speed aerodynamics may be summarized as follows:

- (1) Cost is low compared with wind-tunnel or free flight tests.
- (2) Visual observation of flow in the transonic region is possible since the test model can pass through the sonic range with relative slowness and still avoid choking phenomena.
- (3) Various features of the flow such as shock wave formation, vortices, and turbulence, may be observed and photographed.
- (4) High supersonic Mach numbers are simulated at speeds of a few feet per second.

---

<sup>5</sup>Bruman, J.R., "Application of the Water Channel Compressible Gas Analogy." (North American Aviation Incorporated Engineering Report, No. NA-47-48, 1947).

## THEORY

The theory of the hydraulic analogy as given by Preiswerk<sup>1</sup> will be reviewed here.

The basic assumptions made in the mathematical development of the hydraulic analogy are:

- (1) The flow is irrotational.
- (2) The vertical accelerations at the free surface are negligible compared to the acceleration of gravity. By this assumption the pressures at any point in the fluid depend on only the height of the free surface above that point.
- (3) The flow is without viscous loss, and is uniform along any vertical line.

Throughout the following discussion of the development of the hydraulic analogy the symbol  $d_0$  represents the total head (water depth for  $V = 0$ ), while the symbol  $d$  represents any water depth in the flow field. In the discussion of the compressible gas equations the symbols  $T_0$ ,  $\rho_0$ , and  $p_0$  represent values in the undisturbed gas, and  $T$ ,  $\rho$  and  $p$  represent values in the disturbed flow field. Other symbols used are as follows:

## List of Symbols

$c_p$	Specific heat at constant pressure
$c_v$	Specific heat at constant volume
$\gamma$	Adiabatic gas constant, ratio of $c_p$ to $c_v$ of gas

---

<sup>1</sup>Preiswerk, Ernst., "Application of the Methods of Gas Dynamics to Water Flows With Free Surface."

Part I. "Flows With No Energy Dissipation," NACA TM No. 934, 1940.



T	Absolute temperature of gas
$\rho$	Mass density of gas
p	Pressure of gas
h	Enthalpy or total heat content
q	Dynamic pressure of gas
$\sigma$	Surface tension of liquid
P	Pressure coefficient $\frac{p - p_s}{q}$
$\mu$	Viscosity of liquid
a	Speed of sound in gas $\sqrt{\gamma \frac{p}{\rho}}$
a*	Critical velocity. (Velocity at which $M = 1$ ).
V	Velocity of flow
d	Water depth
M	Mach number, stream value unless otherwise indicated ( $\frac{V}{a}$ for gas; $\frac{V}{gd}$ for water)
g	Acceleration of gravity
$F_c$	Compressibility factor
x, y	Rectangular coordinate axes
u, v	Components of velocity in x and y directions, respectively
$\phi$	Velocity potential in two-dimensional flow
$\lambda$	Wave length of surface waves in fluid
U	Velocity of propagation of surface waves in fluid
R	Reynolds number $\frac{(Vc\rho)}{(\mu)}$
$\bar{u}, \bar{v}$	Nondimensional velocity components in x and y directions, respectively. (Reference velocity $a^*$ ; in hydraulic jump $a_1^*$ , the critical velocity before the jump. $\bar{u} = \frac{u}{a^*}, \bar{v} = \frac{v}{a^*}$ )

#### Subscripts

No subscript any value of variable

o value at stagnation ( $V = 0$ )

$l$	local value of variable; value at surface of model, at channel walls, or in fields of flow
$s$	value in undisturbed streams
$\max$	maximum value of variable
$x$	partial derivative with respect to $x$ i. e. $\phi_x = \frac{\partial \phi}{\partial x}$ ; $\phi_{xx} = \frac{\partial^2 \phi}{\partial x^2}$
$y$	partial derivative with respect to $y$
1	Conditions before the hydraulic jump
2	Conditions after the hydraulic jump

The development of the analogy between the flow of water with a free surface and the flow of a compressible gas may be obtained by setting up the energy equations for the two flows.

For water, the energy equation gives

$$v^2 = 2g(d_0 - d) \quad \text{and}$$

$$v_{\max} = \sqrt{2gd_0}$$

In a gas the energy equation gives

$$v^2 = 2gc_p (T_0 - T) \quad \text{and}$$

$$v_{\max} = \sqrt{2gc_p T_0}$$

By equating the ratio of  $\frac{v}{v_{\max}}$  for water to the ratio of  $\frac{v}{v_{\max}}$  for gas it is seen that

$$\frac{d_0 - d}{d_0} = \frac{T_0 - T}{T_0} \quad \text{or}$$

$$\frac{d}{d_0} = \frac{T}{T_0}$$

It is seen, therefore, that the ratio of the water depth corresponds to the ratio of the gas temperatures.

By a comparison of the equations of continuity for the gas and the liquid a further condition for the analogy may be derived.

The continuity equation for water is

$$\frac{\partial (ud)}{\partial x} + \frac{\partial (vd)}{\partial y} = 0$$

For the two dimensional gas flow the equation of continuity is

$$\frac{\partial (u)}{\partial x} + \frac{\partial (v)}{\partial y} = 0$$

It is seen that the continuity equations of the two flows have the same form, and from the two equations a further condition for the analogy may be derived

$$\frac{d}{d_0} = \frac{\rho}{\rho_0}$$

This relationship shows that the ratio of the local density  $\rho$  to stagnation density  $\rho_0$  is analogous to the ratio of the local depth  $d$  to the stagnation depth  $d_0$ . It is assumed that the flow is steady and two dimensional. For the liquid flow it is assumed that the flow is steady and bounded by a free surface on the top and a flat surface on the bottom, and that the liquid is incompressible.

It has been shown that the water depth is analogous both to the mass density of the gas and to the temperature of the gas. These relationships can only exist if an assumption is made in regards to the nature of the gas. The analogy requires that for the compressible gas,  $\gamma = 2$ . This relationship is shown as follows:

For adiabatic gas flow

$$\frac{\rho}{\rho_0} = \left( \frac{T}{T_0} \right)^{\frac{1}{\gamma-1}}$$



Since the hydraulic analogy requires that  $\frac{p}{p_0} = \frac{d}{d_0}$  and  $\frac{T}{T_0} = \frac{d}{d_0}$  it is obvious that  $\frac{d}{d_0} = \left(\frac{d}{d_0}\right)^{\frac{1}{\gamma-1}}$  and the equation is satisfied only when  $\gamma = 2$ .

From the adiabatic relation

$$\frac{p}{p_0} = \left(\frac{d}{d_0}\right)^{\gamma}$$

it is seen that, since  $\gamma=2$ ,

$$\frac{p}{p_0} = \left(\frac{d}{d_0}\right)^2$$

The differential equation for the velocity potential of the water flow is

$$\phi_{xx} \left(1 - \frac{\phi_x^2}{gd}\right) + \phi_{yy} \left(1 - \frac{\phi_y^2}{gd}\right) - 2\phi_{xy} \frac{\phi_x \phi_y}{gd} = 0$$

The differential equation for the velocity potential of a two dimensional compressible gas flow is

$$\phi_{xx} \left(1 - \frac{\phi_x^2}{a^2}\right) + \phi_{yy} \left(1 - \frac{\phi_y^2}{a^2}\right) - 2\phi_{xy} \frac{\phi_x \phi_y}{a^2} = 0$$

The two equations become identical when  $\frac{gd}{2gd_0} = \frac{a^2}{2gh_0}$

That the velocity  $\sqrt{gd}$  is the velocity of propagation of surface waves (small disturbances) in shallow water may be shown as follows:

Page<sup>2</sup> gives the following equation for the velocity of water waves under the action of gravity and surface tension:

$$v = \left[ \left( \frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{f\lambda} \right) \tanh \frac{2\pi d}{\lambda} \right]^{1/2}$$

Inspection of the equation shows that as wave length becomes great in comparison with the water depth ( i.e.  $\frac{d}{\lambda} \rightarrow 0$ ,  $\tanh \frac{2\pi d}{\lambda} \rightarrow \frac{2\pi d}{\lambda}$ )

---

<sup>2</sup> Page, Leigh, Introduction to Theoretical Physics, New York: D. Van Nostrand Co. Inc., 1935, p.250

and if the surface tension is equal to zero, the wave velocity becomes

$$V = \sqrt{gd}$$

Since the velocity of sound,  $a$ , is the velocity at which small disturbances are propagated in gas flow, the ratio  $\frac{V}{\sqrt{gd}}$  for the shallow water flow is analogous to the Mach number  $\frac{V}{a}$  in the gas flow.

The local Mach numbers about the model are determined from

$$M_\ell = \left[ \frac{2 (d_o - d_\ell)}{d_\ell} \right]^{\frac{1}{2}}$$

which is obtained by substituting the equation for the local velocity in the expression for Mach number.

The pressure coefficient for the water flow may be determined as follows:

For a compressible gas<sup>3</sup>

$$P = F_c \left[ \frac{\frac{p_\ell}{p_s} - 1}{\frac{p_o}{p_s} - 1} \right]$$

When the analogous relationships for water are substituted in the equation for the pressure coefficient the resulting equation is

$$P = F_c \left[ \frac{\frac{d_\ell^2}{d_s^2} - 1}{\frac{d_o^2}{d_s^2} - 1} \right]$$

---

<sup>3</sup> Orlin, James., Norman J. Linder and Jack G. Bitterly, "Application of the Analogy Between Water Flow with a Free Surface and Two-Dimensional Compressible Gas Flow," NACA TN, No. 1185, 1947. p.8

For a compressible gas the compressibility factor<sup>4</sup> is

$$F_c = \frac{2}{\gamma M_s^2} \left[ \left( 1 + \frac{\gamma-1}{2} M_s^2 \right)^{\frac{\gamma}{\gamma-1}} - 1 \right]$$

Since the hydraulic analogy requires that  $\gamma = 2$  the compressibility factor for the water flow is

$$F_c = 1 + \frac{1}{4} M_s^2$$

It may be noticed that no theory has been presented concerning shock waves or supersonic flow.

In the hydraulic analogy a compressible shock in the gas is analogous to a hydraulic jump in the water flow. The hydraulic jump may be defined as an unsteady motion in which the velocity of flow may strongly decrease for short distances, and the water depth suddenly increase. Hydraulic jumps occur in water whose velocity of flow is greater than the wave propagation velocity. Preiswerk developed the relationships between conditions in front of the shock wave and behind the shock wave for the hydraulic analogy. In developing the theory the assumption is made that the water motion is entirely unsteady which indicates that the water jumps suddenly along a line from the lower water level to the level after the jump. The assumptions originally made are also carried through the discussion of the shock wave analogy.

For the water flow without a jump the energy equation holds

$$V^2 = 2g(d_0 - d)$$

where  $d_0$  the total head is a constant. After a hydraulic jump has taken place the total head  $d_{02}$  is smaller than  $d_{01}$ , since part of the kinetic

---

<sup>4</sup>LoC. Cit.



energy of the water is converted into heat, and is treated as lost energy. For the flow after the jump the energy equation again applies and now becomes

$$V_2^2 = 2g(d_{o_2} - d)$$

To solve this equation for  $d_{o_2}$ , the new total head, use is made of the shock polar diagram to determine  $V_2$ , the water velocity behind the shock wave.

The equation for the shock polar for the hydraulic analogy is

$$\bar{V}_2^2 = [\bar{u}_1, -\bar{u}_2] [\bar{u}_2 - u_1 \cdot \sqrt{\frac{3 - \bar{u}_1^2}{(3 - 4\bar{u}_1\bar{u}_2 + 3u_1^4)}}]$$

By varying the parameter  $u_1$ , and solving for  $\bar{V}_2$  in terms of  $\bar{u}_2$ , a family of curves will result. The curves are similar to the shock polars of an ideal gas, but they show one characteristic difference. For the maximum velocity both cases become circles. However, for water the circle passes through the origin while for a perfect gas the circle does not pass through the origin. The shock polars for water were constructed for the speeds investigated in this thesis and are presented in Figures 5 and 6.

In Figure 7 shock polars are drawn for air and for water for the same initial flow conditions. It may be seen that for continually decreasing shocks the polars approach each other.

The energy loss in the flow with hydraulic jumps is developed to be

$$\Delta e = 1 - \frac{h_{o_2}}{h_{o_1}}$$

This is the relative energy converted into heat. However, for gas the rise in heat during the shock is not lost energy since the total heat

content remains the same before and after the shock. Curves of constant energy for water flow are presented by Preiswerk. From the curves it may be seen that the energy loss is small over a large region. For the velocities investigated in this thesis the relative loss is less than one per cent. Because of this small shock loss the analogy between the two types of flow is still satisfied.

By employing the shock polar diagrams and the equations given by Preiswerk conditions across a shock wave may be determined.

The various quantities which may be obtained from a shock polar diagram are illustrated in Figure 7. No further explanation will be given, since the theory and development of shock polar diagrams is well covered in appropriate textbooks.<sup>5</sup>

A summary of the analogous relationships is given below.

Two-Dimensional compressible gas flow $\gamma = 2$	Incompressible liquid flow (water)
Temperature Ratio, $\frac{T}{T_0}$	Water depth ratio, $\frac{d}{d_0}$
Density ratio, $\frac{\rho}{\rho_0}$	Water depth ratio, $\frac{d}{d_0}$
Pressure ratio, $\frac{p}{p_0}$	Square of water-depth ratio $\left(\frac{d}{d_0}\right)^2$
Velocity of sound, $a = \sqrt{\frac{\gamma p}{\rho}}$	Wave velocity, $\sqrt{gd}$
Mach number, $\frac{V}{a}$	Mach number $\frac{V}{\sqrt{gd}}$
Shock wave	Hydraulic jump

<sup>5</sup>Liepmann, Hans Wolfgang, and Allen E. Puckett, Introduction to Aerodynamics of a Compressible Fluid. (New York: John Wiley and Sons, Inc., 1947 p. 53)



## EQUIPMENT

In designing the equipment for conducting experiments with the hydraulic analogy, two methods of approach were available. These methods were:

- (1) The model was to remain stationary with the water flowing past the model.
- (2) The water was to remain stationary with the model moving through the water.

The first type of channel would necessarily be the more elaborate, and therefore, the more expensive. It would be difficult to change Mach number or to have decelerating flow or accelerating flow. The presence of a boundary layer on the bottom and sides of the channel would further complicate a channel of the first type. However, with the model remaining stationary it would be easy to measure the water depth about the model, which is very important in the hydraulic analogy.

A channel of the second type results in an easy measurement of speed and acceleration, and therefore, Mach number. The big disadvantage of this type of channel is the difficulty of measuring the water depth about the model, since the model moves relative to the channel.

Taking all factors into consideration, it was decided to construct a channel of the second type, Figure 1.

The water channel frame work was made of structural steel bolted together. The channel is four feet wide and twenty feet long. The channel bottom is plate glass one quarter of an inch thick in two five foot sections and one ten foot section. The ten foot section of glass is located at the "test" end of the channel. The glass bottom is supported by transverse members at thirty inch intervals. The purpose of

the glass bottom is to provide a smooth surface for the models to slide upon, and to provide for a method of photographing the flow of water about the models. The channel bottom was maintained to a level of .01 inch by a screw-jack type device on each of the channel's supporting legs. The accuracy of the channel bottom level was determined by using a surveyor's transit.

A drain is provided at one end of the channel.

The model carriage is made of steel tubing welded together, and travels the entire length of the channel on a track which is located along each side of the channel, Figure 2. The carriage travels on four rubber-tired, ball bearing wheels. Four additional wheels restrain the carriage in a horizontal plane. Safety stops are provided on the carriage track to prevent the carriage and model from running off either end of the channel.

The carriage is driven by a reversible, direct current motor through a set of reduction gears and a continuous cable. The motor is rated at 19.5 amperes, 24 volts. A direct current motor is used to provide a high starting torque in order to quickly accelerate the carriage to the desired speed. The driving cable is  $3/32$  inch diameter conventional aircraft steel cable. The cable goes one and one-half times around the driving pulley, and is rigged with about twenty pounds tension. The speed of the carriage drive motor is controlled by a 6.7 amperes, 18 ohm rheostat. The rheostat has a calibrated dial with settings to give carriage speeds from one to ten feet per second. By proper operation of the rheostat the carriage may be run at increasing, decreasing, or constant speeds. After



experimenting with the water channel it was observed that the direct current motor did not result in constant carriage speeds below a speed of one foot per second. Consequently, a "Speed-Ranger" constant speed device powered by one-quarter horse power, three phase, alternating current, motor was installed. The "Speed-Ranger" device resulted in carriage speeds over the range of from .50 feet per second up to one foot per second. The "Speed-Ranger" speed control serves as an alternate drive device since the direct current motor was found to be more convenient for high speed runs, and for accelerating or decelerating runs. A photograph of the drive mechanisms is shown in Figure 3.

An observation platform is located at the channel test section. Mounted on the platform is a control panel which includes all switches and controls for operating the water channel. The control panel and observation platform are so set up that only one person is required to operate the water channel during a testing program. Included on the platform are mounts for the cameras used to photograph the water flow about the models.

The arrangement of the equipment for photographing the flow about the models is shown schematically in Figure 4. Two cameras are used to obtain the flow photographs. One camera, mounted on the platform floor, photographs the local water depth about the models, while the second camera, mounted on a boom extending out over the channel, photographs the wave formation caused by the models.

As seen in Figure 4, the camera used to photograph the flow about the models in the horizontal plane projects through a diffusion screen located approximately four feet above the water surface. A second diffusion screen is located immediately under the glass in the area to be

photographed. The lighting arrangement consists of a boom spotlight directed on the model from above and to the rear of the model, and a large spotlight directed on the diffusion screen above the test section. Four flood lights illuminate a cellox screen located on the floor beneath the channel. The four flood lights result in an even lighting effect over the test area.

The spotlights result in the wave formations casting shadows on the screen directly under the glass, and these shadows are then photographed. An Eastman Medalist Camera with a setting of one four-hundredth of a second at  $f: 7.5$  lens opening is used to photograph the wave formations. The camera shutter was tripped electrically at the correct time by a cam device, mounted on the model carriage, activating a microswitch located on the channel track. The results of using the photographic technique here described are entirely satisfactory.

From the development of the theory of the hydraulic analogy it may be seen that the water depth distribution about the model is very important if a quantitative analysis of the flow is to be made. To determine the water depth about the model, a Speed Graphic camera is used to photograph the water depth distribution in the vertical plane. The camera is mounted on the platform above the water level to insure no interference from waves between the model and the camera. The model is illuminated by a 500 watt slide projector. The projector is so masked that at the time the camera is tripped only the model is illuminated. This feature was found necessary because any extensive side lighting interfered with the horizontal flow photographs. The camera is tripped by the same switch and cam arrangement that operates the overhead camera. The best camera



setting was found to be one four-hundredths of a second at f: 3.7 lens opening.

The airfoils chosen for testing in the water channel are N. A. C. A. airfoils developed by the National Advisory Committee for Aeronautics, having the following designations:<sup>1</sup>

(1) NACA 2S-(50) (03)-(50) (03) circular arc airfoil

(2) NACA 1S-(30) (03)-(30) (03) diamond airfoil.

The designation of the circular arc airfoil indicates that the maximum thickness of both the upper and lower surface is located at the fifty percent chord point, and that the maximum thickness is six percent of the chord length. The designation of the diamond shaped airfoil shows that the maximum thickness of the upper and lower surface is located at the thirty percent chord point, and that the total thickness is six percent of the chord. These particular airfoils were chosen because they are basic supersonic airfoils.

The airfoil models are made of wood. Each model has a twelve inch chord. The models are painted a flat white on the bottom and sides while the top is painted a flat black. This paint scheme was selected to provide good photographic properties for the flow pictures. A single black line was ruled on each side of the models from the leading edge to the trailing edge. The line is used as a reference line in the determination of the water depth about the models, since the water depth is scaled from photographs of the flow in the vertical plane. Figure 8 and Figure 9 show the model dimensions.

---

<sup>1</sup>Lindsey, W.F., Bernard N. Daley, and Milton D. Humphreys, "The Flow and Force Characteristics of Supersonic Airfoils at High Subsonic Speeds." U. S. National Advisory Committee for Aeronautics Technical Note, No. 1211, 1947. p.3.

Since the hydraulic analogy applies only to two dimensional flow the airfoil models actually slide on the glass bottom of the channel to cause the required flow. The models are attached to the carriage by a linkage as shown in Figure 2.

The angle of attack of the models is determined by positioning the leading edge and the trailing edge in relation to the channel sides. To facilitate the setting of each angle of attack a calibration curve was drawn giving the distances of the leading edge and trailing edge from the channel sides for each desired angle. This curve is presented in Figure 10.

### PROCEDURE

The procedure for model testing in the water channel consisted of running the two-dimensional airfoil models through the water in the channel at predetermined speeds. The water depth for all tests was one quarter of an inch while the angle of attack of the models in each case was zero degrees. The time for the carriage and model to move across the test section was recorded by an electric timing clock which was started by a cam device on the model carriage actuating a micro-switch located on the carriage track. At the end of the three foot run the cam released the micro-switch and stopped the clock. The model velocity could then be calculated.

Simultaneous pictures were taken of the wave formations set up by each model and of the local water depth distribution about the models. A complete discussion of photographic technique involved is included in the previous chapter on equipment.



## DISCUSSION

Before the hydraulic analogy may be applied to the investigation of problems of compressible flow, the flow pictures must be properly interpreted.

From an inspection of the flow photographs in the horizontal plane a series of alternately dark and light bands of decreasing intensity may be seen starting at the leading edge and trailing edge of each model. A similar condition also exists at the section of maximum thickness of the diamond shaped airfoil. The problem immediately arises as to which part of the pattern directly applies to the hydraulic analogy? Of major importance is the determination of that part of the flow pattern which represents a hydraulic jump or shock wave. Waves on the water surface act as lenses in that they divert the light rays in the direction of increasing water depth. Therefore, the sharp bright lines represent that part of each wave or ripple which has the maximum convex curvature. The dark area preceding the sharp white lines represents the remainder of each wave. It may be then concluded that a shock wave is recorded on the photographs as a white line preceded by a dark area.

In addition to, and in front of, the shock waves the flow photographs show ripples or capillary waves. The capillary waves are not part of the hydraulic analogy since they are purely water waves and depend primarily on the surface tension of the liquid. Figure 11 shows the relation of wave propagation velocity to wave length for a water depth of one quarter of an inch. Disturbances of wave lengths less than that corresponding to the minimum wave velocity are capillary waves.

The refraction pattern formed by a hydraulic jump and capillary waves are shown in Figure 12. From these patterns a qualitative study of the photographs may be made.

It was found by North American Aviation Incorporated<sup>1</sup> that expansion waves in a water channel do not agree with the theory of the hydraulic analogy, but rather tend to travel at a constant speed not dependent on the static water depth, or the model speed. This fact is borne out by an examination of the diamond airfoil flow photographs in the horizontal plane. At the point of maximum thickness where an expansion takes place there is no definite or consistent pattern to indicate an expansion wave. However, from the photographs showing the water depth distribution about the diamond airfoil a definite level drop may be seen at the change in section. From the side photographs, then, an expansion wave may be detected.

A comparison was made between the shock wave angles obtained by measurement from the flow pictures and the shock wave angle as obtained from the shock polars for each speed investigated. A further comparison was made with the results obtained by using the shock wave charts presented by Laitone.<sup>2</sup> The results are given in Figures 25 and 26. It may be seen that the three methods of obtaining the shock wave angles for a given free stream Mach number and semi-wedge angle agree very closely. This close agreement indicates that the application of the

---

<sup>1</sup>Bruman, J.R., "Application of the Water Channel Compressible Gas Analogy." North American Aviation Incorporated Engineering Report, No. NA-47-87, 1957.

<sup>2</sup>Laitone, E.V., "Exact and Approximate Solutions of Two-Dimensional Oblique Shock Flow." Journal of the Aeronautical Sciences, Vol. 14, No.1, January, 1947, pp. 25-41.



hydraulic analogy is a useful tool in the study of shock wave formation.

To insure no distortion as a result of enlarging the flow pictures, a comparison was made between contact prints of each wave and enlargements of the same wave formation. No distortion, as a result of enlargement, was noted.

Curves showing the variation of local Mach number about the airfoil models are presented in Figure 27 and Figure 28. The local Mach number variation for each model was calculated from the  $M_l$  equation<sup>3</sup>. It may be observed that the local Mach number is a function of the total head and the local water depth. Since the total head consists of a velocity head and a static head the shock polar diagrams were employed to find the water velocity behind the shock wave, and the static head was determined from the side photographs. The static head was taken as that water depth where the water line on the model was horizontal. The local water depth was obtained by measurement from the side photographs.

Airfoil theory predicts that for a diamond airfoil in a supersonic flow at zero degrees angle of attack the local Mach number remains constant over the forward part of the airfoil up to the point of maximum thickness, at which time it increases abruptly to a new value. From the curves of local Mach number distribution for the diamond airfoil it may be seen that the local Mach number is not constant over the forward thirty percent of the airfoil chord, but goes to relative high values at the leading edge. The curve does not change abruptly at the point of maximum thickness, as theory predicts, but there is a smooth transition

---

<sup>3</sup> Cf Ante, p.10

from the lower value to the high value as the expansion takes place. The local Mach number then becomes constant over the remainder of the airfoil.

The high values of the Mach numbers at the leading edge may be explained since in the development of the hydraulic analogy it was assumed that the hydraulic jump took place along a single line, and the water depth changed abruptly along that line. Actually, these conditions did not realize since the hydraulic jump required a finite distance relative to the airfoil in which to form. It therefore appears that the hydraulic analogy will not give the correct local Mach number distribution immediately following the leading edge for the diamond airfoil.

For the circular arc airfoil an entirely different trend of the local Mach number curve may be observed. As for the diamond airfoil, the local Mach number goes to a relative high value at the leading edge. After the minimum value is attained the local Mach number increases over the entire chord length. This distribution of local Mach number is in agreement with theory.

An attempt was made to plot the local Mach number distribution about the models over the range of free stream Mach numbers from .75 to .90. At these slow speeds the meniscus effect resulted in an erratic water depth distribution about the models, and as a result no dependable data was obtained from the side photographs. After this condition was noticed the models were waxed and polished to reduce the surface roughness. No marked improvement in the water depth distribution was noted. It would seem, then, that for slow speed tests deeper water should be used to reduce the relative magnitude of the meniscus effect.



## CONCLUSIONS

- (1) The hydraulic analogy offers a relatively inexpensive method of conducting research in supersonics.
- (2) The water channel is especially suited for class room demonstration of shock wave phenomena. The reflection of shock waves from the channel walls, and the intersection of shock waves may be observed as well as the shock waves formed by various shapes.
- (3) With a more accurate method of depth measurement more quantitative data may be obtained.
- (4) The determination of shock wave angles from the flow photographs for the airfoils tested agrees very closely with results obtained by application of theoretical methods.
- (5) The relative value of the local Mach number distribution about the airfoils tested agree with theory.
- (6) Accurate quantitative results will not be obtained when the hydraulic analogy is applied to the study of air flow because the water flow is analogous to a gas flow with  $M = 2$ , whereas for air  $M = 1.4$ .

## RECOMMENDATIONS

- (1) With the present excellent drive mechanism installed, very accurate model speed is assured. However, the drive device transmits a certain amount of vibration to the model which complicates the flow photographs by setting up excessive secondary ripples. To offset this objectionable vibration the model support linkage should be stiffened. The model itself should be made heavier to help reduce vibration.

(2) The use of fresh water in the water channel results in the hydraulic jump inducing a great number of ripples ahead of itself which has no part in the hydraulic analogy. By allowing the water to remain in the channel over a period of twenty four to forty eight hours results in clearer wave photographs.

(3) By further experimentation with the photographic technique the present large amount of equipment involved could be eliminated except for the underwater lights, and the screen resting on the floor.

(4) The sides of the model should be wet before each run to result in constant meniscus effect.

(5) For test speeds of less than Mach number a water depth somewhat greater than one quarter of an inch should be used in the channel.



## BIBLIOGRAPHY

- Binder, R. C., Fluid Mechanics. New York: Prentice-Hall Inc., 1947. 307 pp.
- Binnie, A. M., and S.G. Hooker, "The Flow Under Gravity of an Incompressible and Inviscid Fluid Through a Constriction in a Horizontal Channel." Proceedings of the Royal Society, Vol. 159, (London, England: 1937), pp. 592-608.
- Bruman, J. R., "Application of the Water Channel Compressible Gas Analogy." North American Aviation Incorporated Engineering Report, No. NA-47-87, 1947.
- Einstein, H.A. and E.H. Baird, "Progress Report on the Analogy Between Surface Waves in Water and Shock Waves in a Compressible Gas." California Institute of Technology Hydraulics Laboratory Progress Report, No. 54, 1946.
- Laitone, E.V., "Exact and Approximate Solutions of Two-dimensional Oblique Shock Flow." Journal of the Aeronautical Sciences, Vol. 14, No. 1.
- Lamb, Horace, Hydrodynamics, 6th Edition, London: Cambridge University Press, 1932, 738 pp.
- Lindsey, W.F., Bernard N. Daley, Milton D. Humphreys, "The Flow and Force Characteristics of Supersonic Airfoils at High Subsonic Speeds." U. S. Advisory Committee for Aeronautics Technical Note, No. 1211, 1947, 15 pp.
- Orlin, James, J. Linder and Jack Bitterly, "Application of the Analogy Between Water Flow With a Free Surface and Two-Dimensional Compressible Gas Flow." U.S. Advisory Committee for Aeronautics Technical Note, No. 1185, 1947, 20 pp.
- Page, Leigh, Introduction to Theoretical Physics, New York: D. Van Nostrand Co., Inc. 1935, 250 pp.
- Preiswerk, Ernst., "Application of the Methods of Gas Dynamics to Water Flows With Free Surface."  
Part I., "Flows with No Energy Dissipation." U.S. National Advisory Committee for Aeronautics Technical Memorandum, No. 934, 1940, 69 pp.  
Part II. "Flows With Momentum Discontinuities (Hydraulic Jumps)." U.S. Advisory Committee for Aeronautics Technical Memorandum, No. 935, 1940, 56 pp.
- Riabouchinsky, D., Mecaniques des Fluides, Comptes Rendu, t. 195 No. 22, Paris, November 28, 1932, pp. 998-999.



Rouse, Hunter, Fluid Mechanics for Hydraulic Engineers. New York and London: McGraw-Hill Book Company, Inc., 1938, 420 pp.

Vennard, John K., Elementary Fluid Mechanics. New York: John Wiley and Sons, Inc. 1947, 339 pp.

## APPENDIX I

## FIGURES

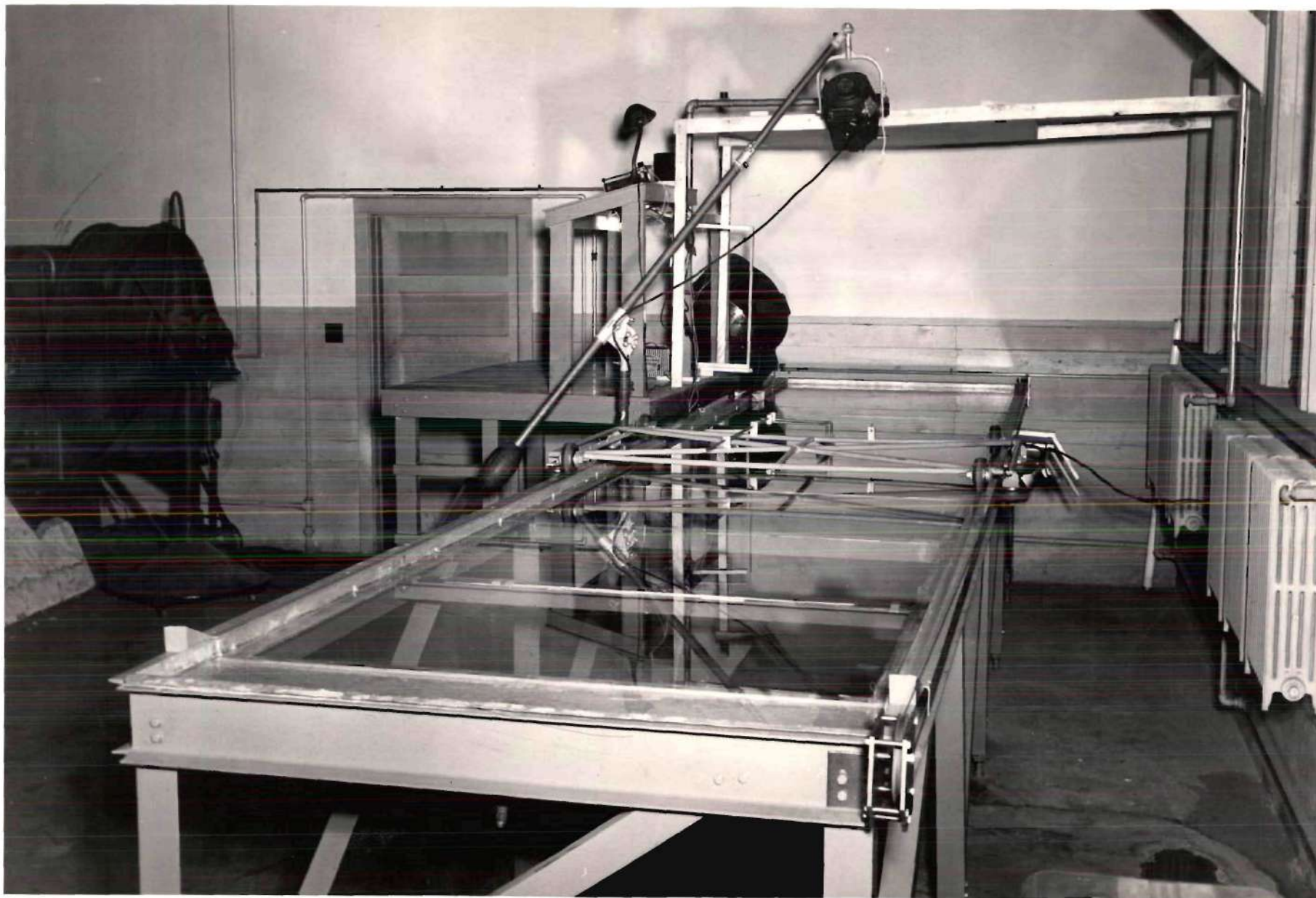


FIGURE 1  
GENERAL VIEW OF WATER CHANNEL



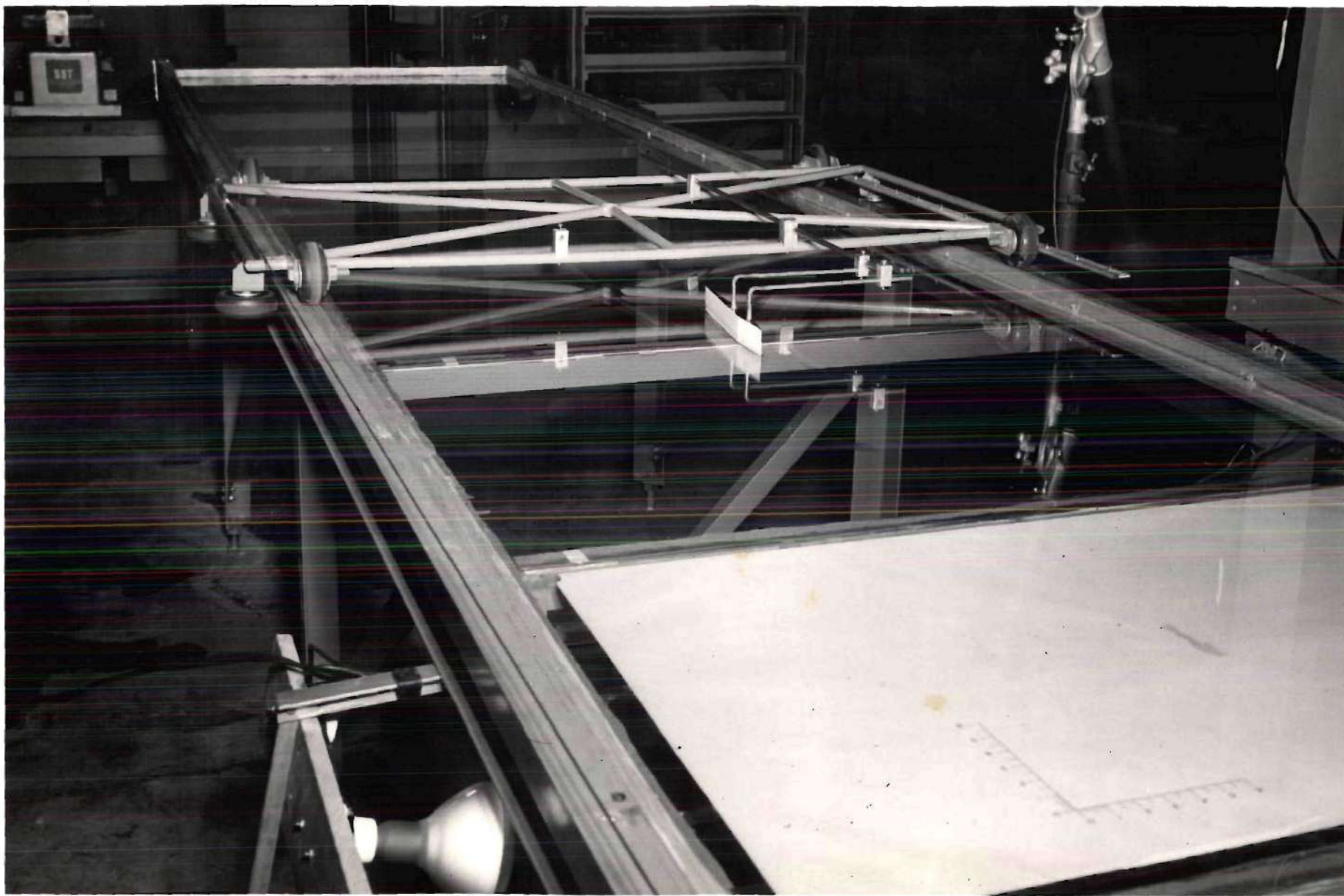
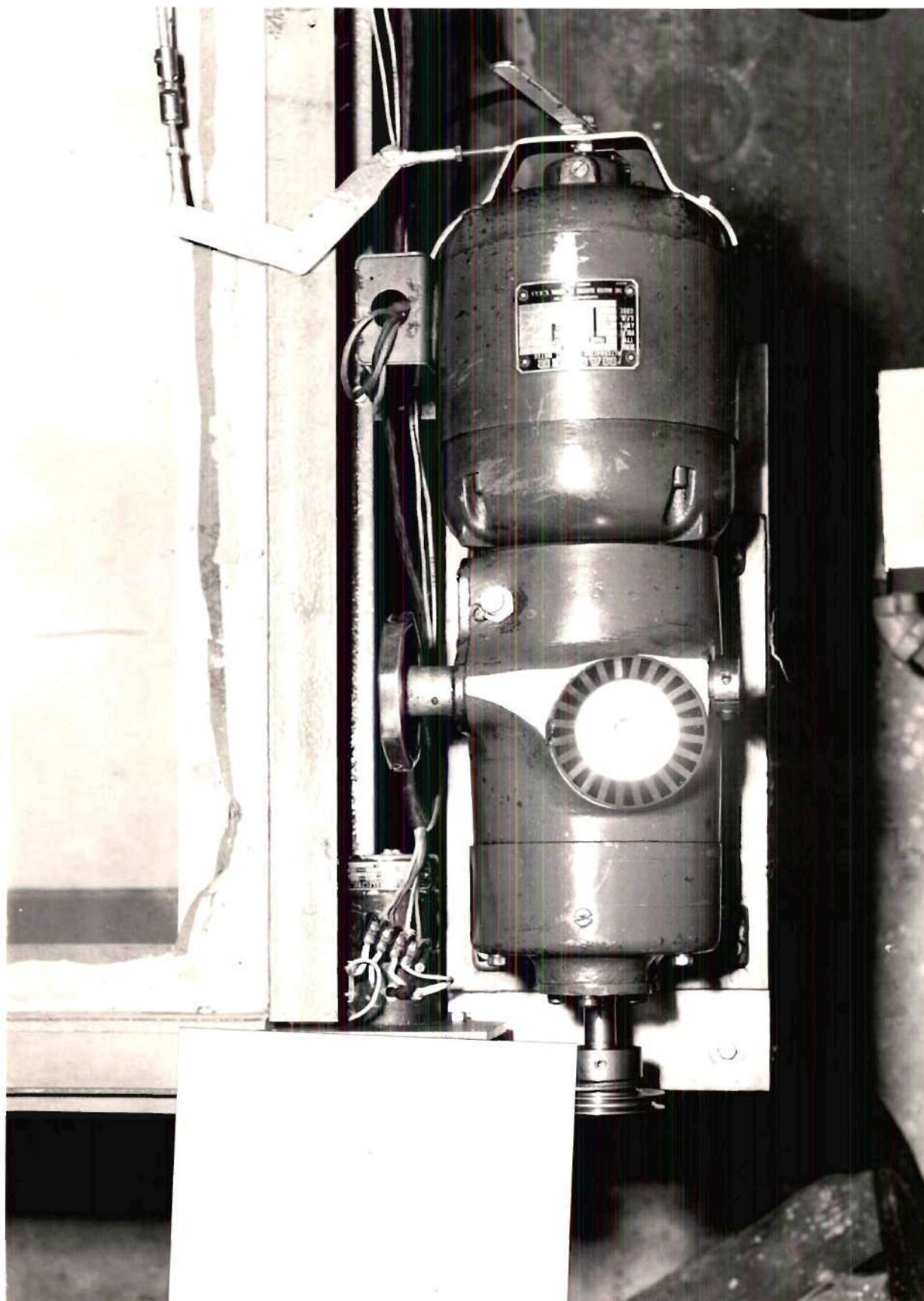


FIGURE 2  
GENERAL VIEW OF WATER CHANNEL



/E MECHANISM

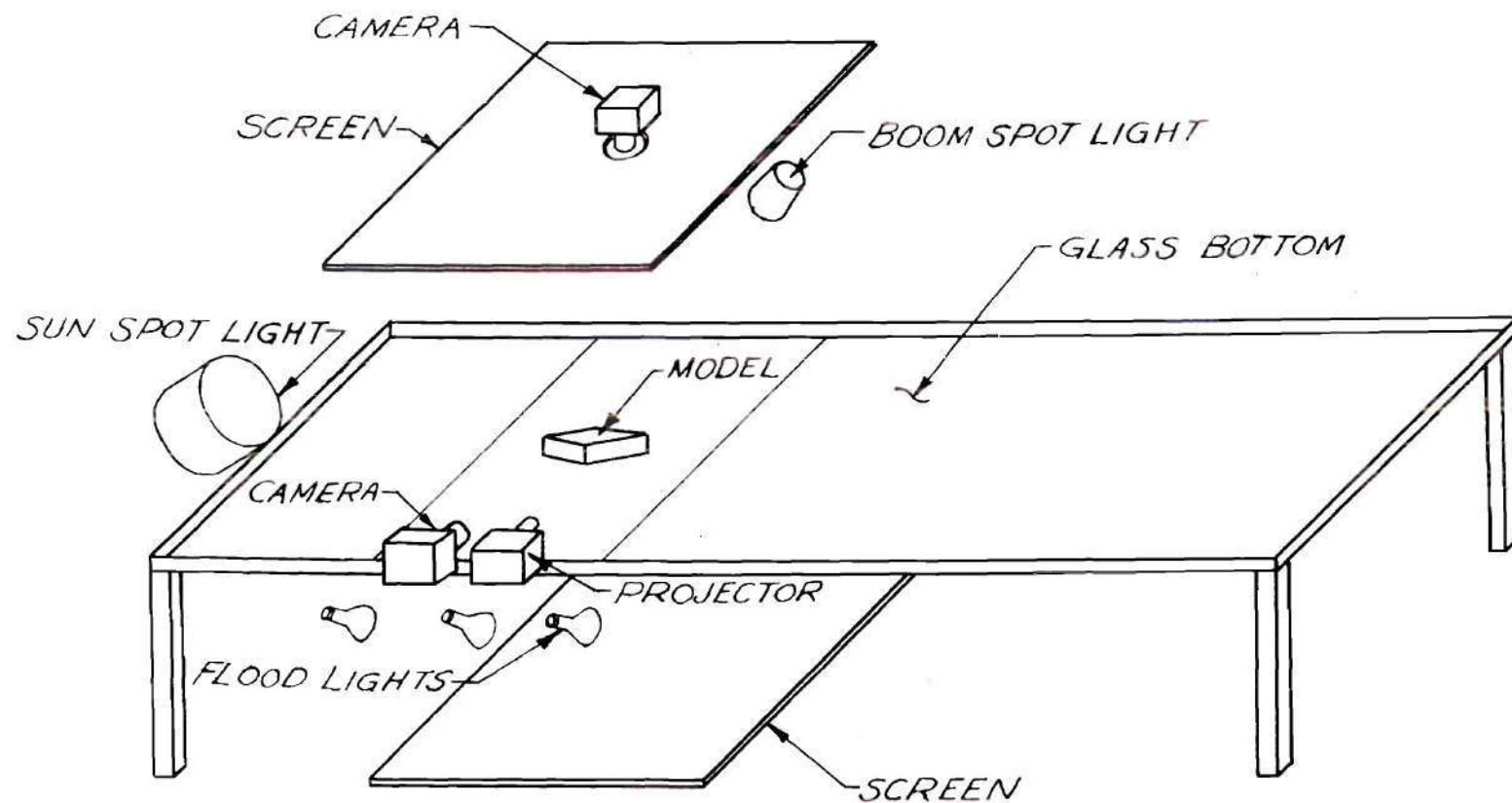


FIGURE 4  
SCHEMATIC DIAGRAM OF WATER CHANNEL AND LIGHTING EQUIPMENT



FIGURE 5  
SHOCK POLAR DIAGRAMS FOR  
DIAMOND AIRFOIL TEST SPEEDS

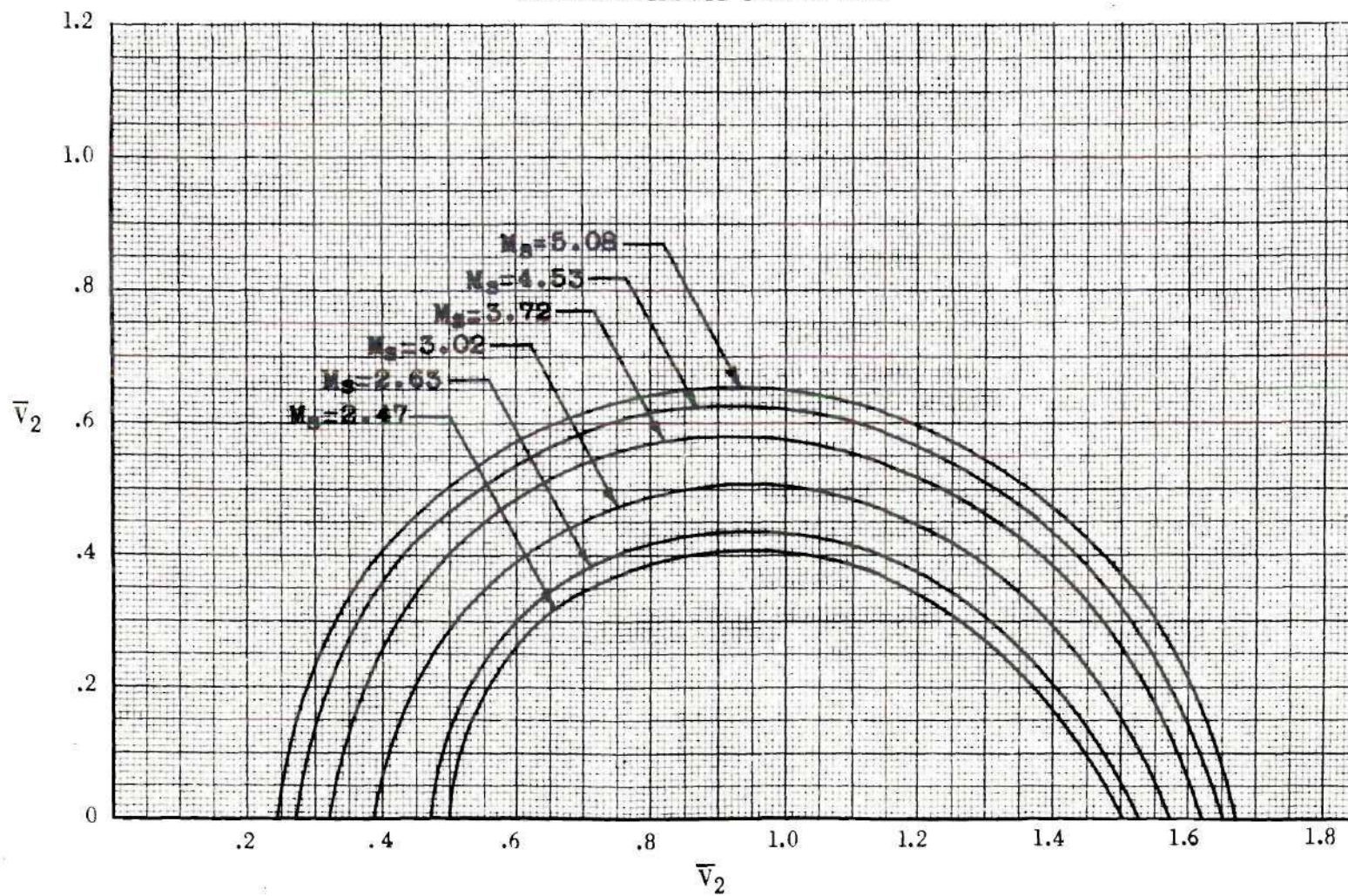




FIGURE 6  
SHOCK POLAR DIAGRAMS FOR  
CIRCULAR ARC AIRFOIL TEST SPEEDS

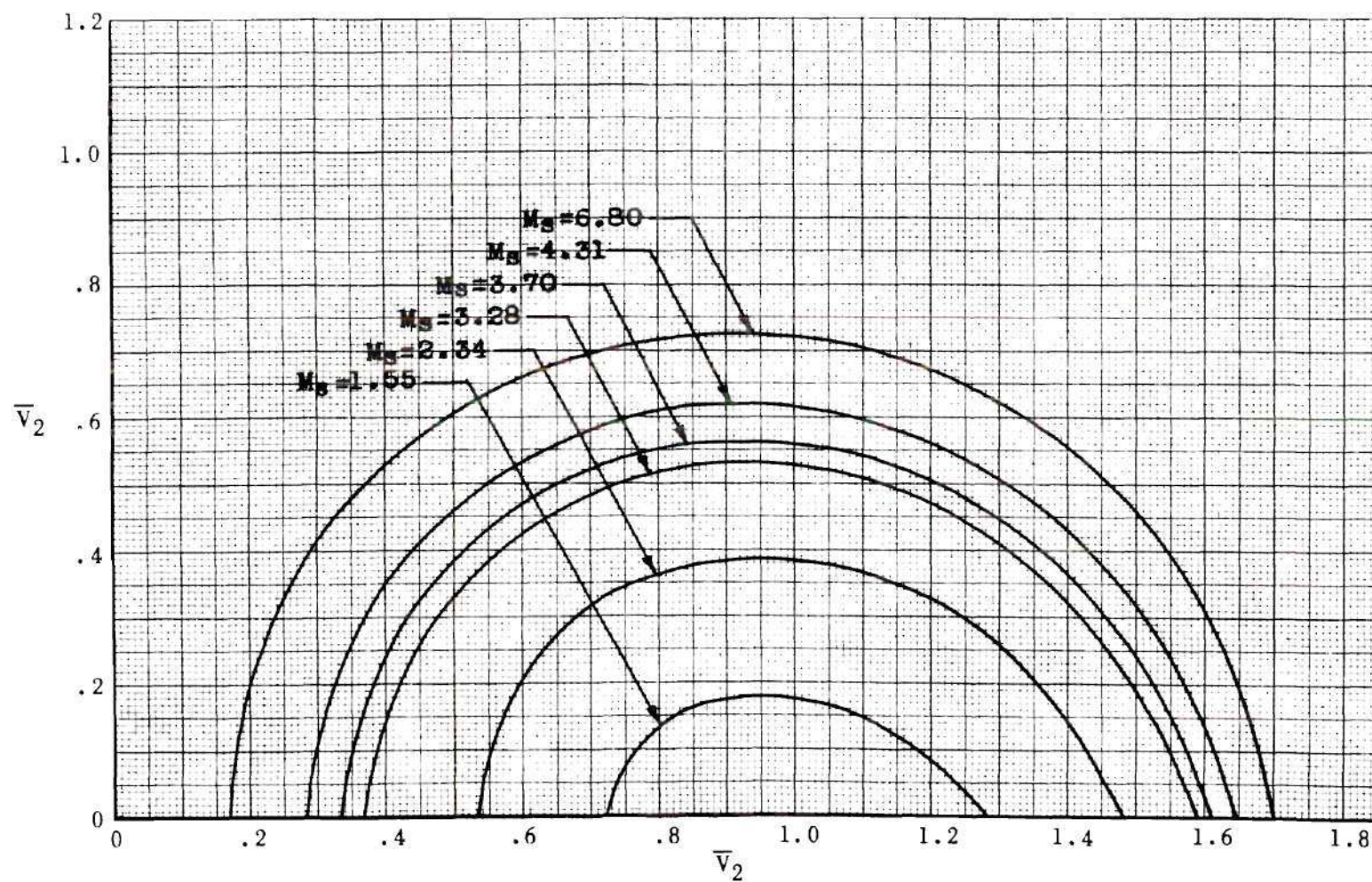
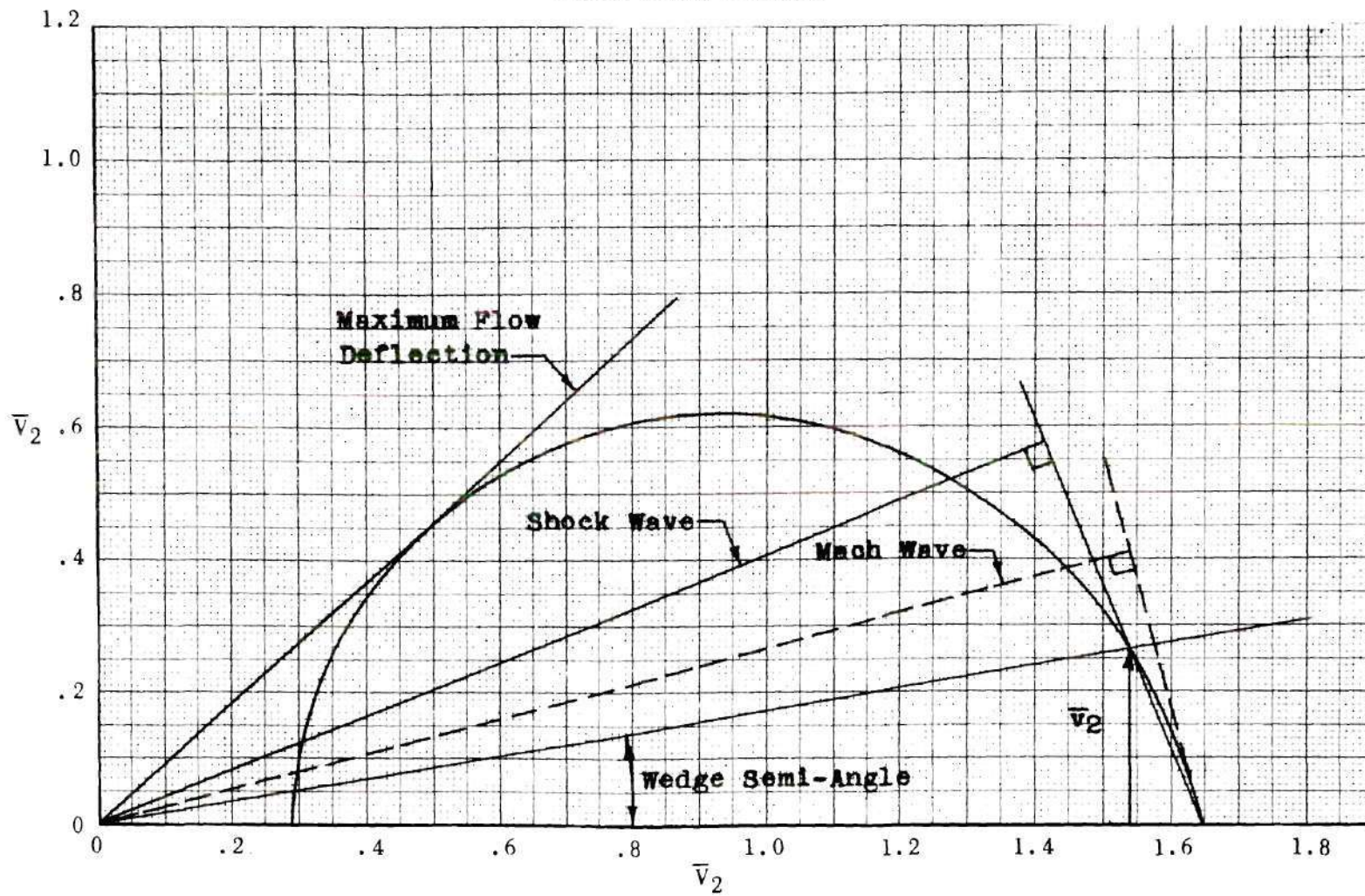
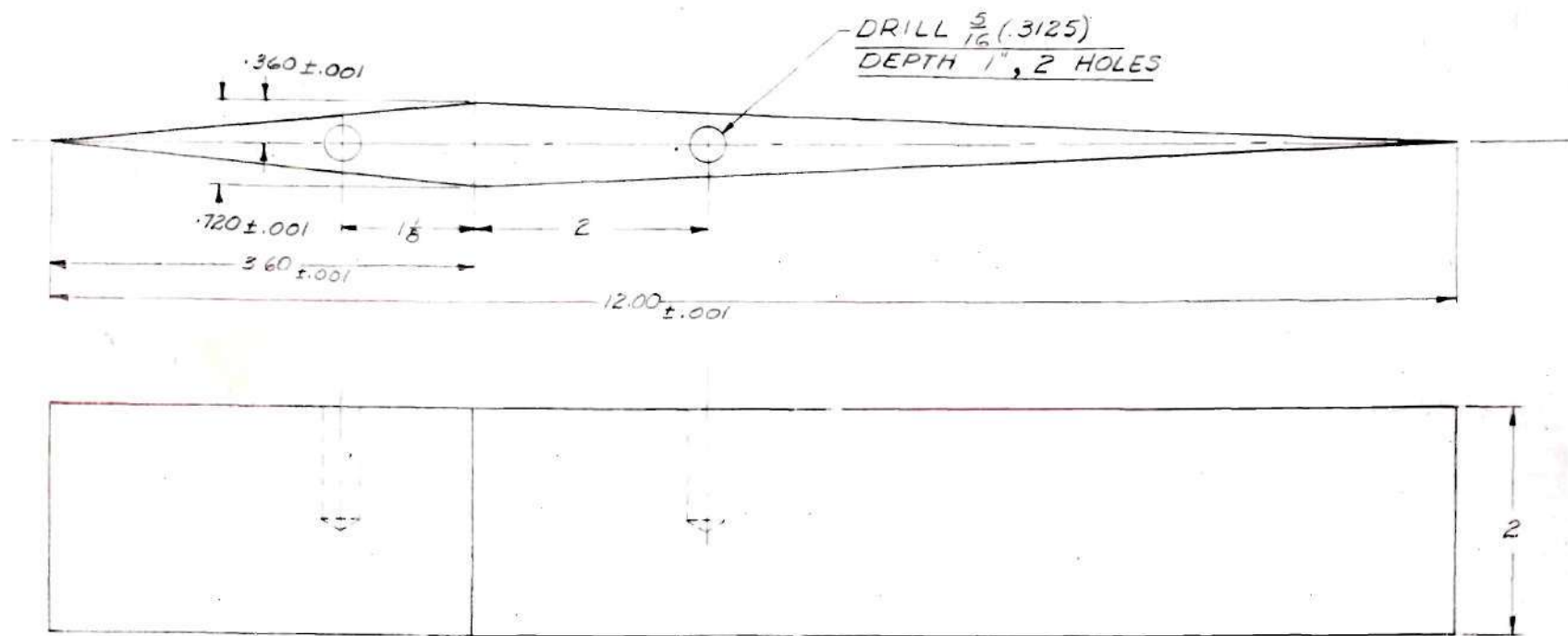


FIGURE 7  
SHOCK POLAR DIAGRAM

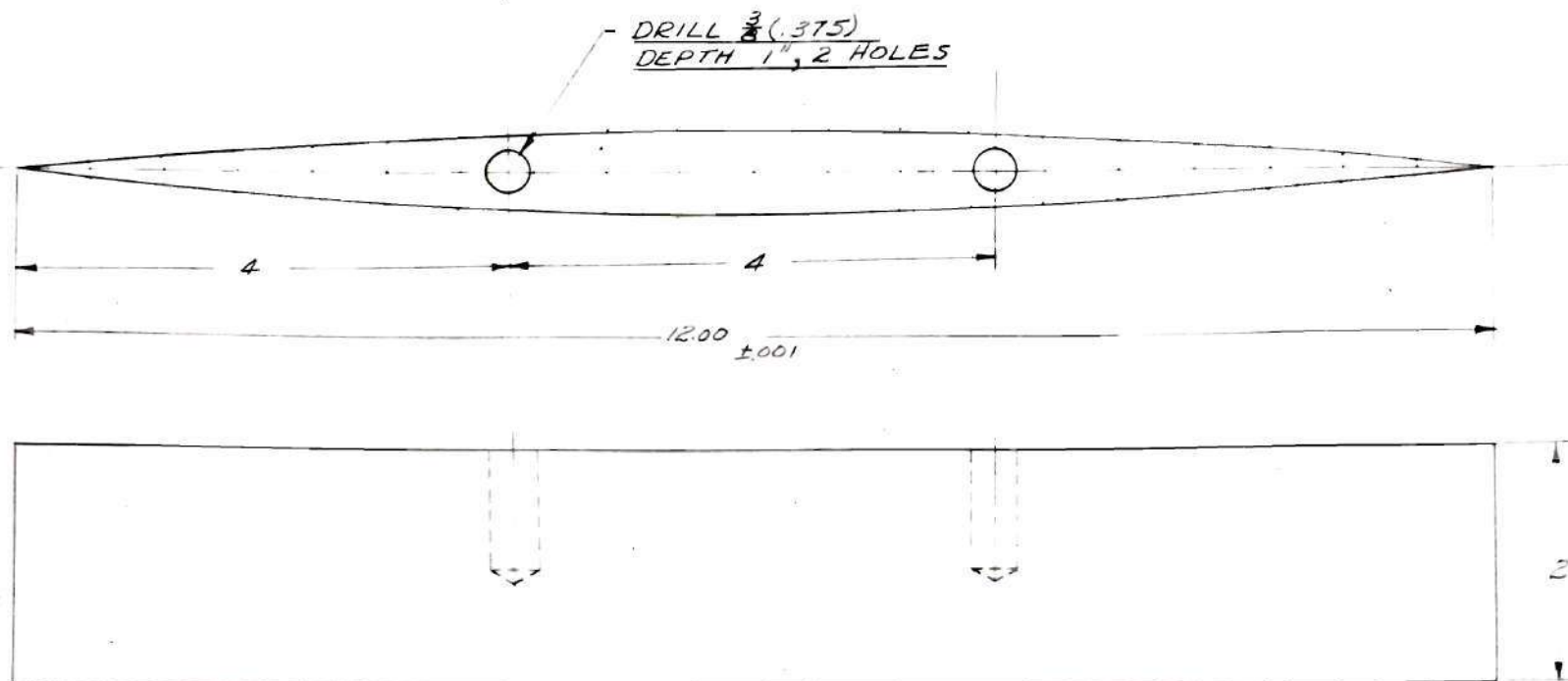






WALNUT	DR. LEADER	APPROVED BY	NO. 100	PROJECT
	HATCH			
FULL	1-11-49	AIRFOIL MODEL FOR WATER CHANNEL TESTING		E/D-521
GEORGE W. GOETTER SCHOOL OF AERONAUTICS GEORGIA SCHOOL OF TECHNOLOGY				

FIGURE 8  
DIAMOND AIRFOIL MODEL



SECTION ORDINATES FOR SYMMETRICAL CIRCULAR ARC AIRFOIL

STATION (IN.)	ORDI- NATE (IN.)	STATION (IN.)	ORDI- NATE (IN.)
0	0	6.6	.356
.6	.068	7.2	.346
1.2	.130	7.8	.328
1.8	.164	8.4	.302
2.4	.230	9.0	.270
3.0	.270	9.6	.230
3.6	.302	10.2	.184
4.2	.328	10.8	.130
4.8	.346	11.4	.068
5.4	.356	12.0	0
6.0	.360		

LEADING EDGE RADIUS = 0  
CIRCULAR ARC RADIUS = 49.944"

WOOD HATCH

FULL 2-16-49 CIRCULAR ARC AIRFOIL FOR WATER CHANNEL TESTING E/D-523

SCHOOL OF AERONAUTICS  
GEORGIA SCHOOL OF TECHNOLOGY

FIGURE 9  
CIRCULAR ARC AIRFOIL MODEL

FIGURE 10  
MODEL AIRFOIL ANGLE OF ATTACK CALIBRATION CURVE

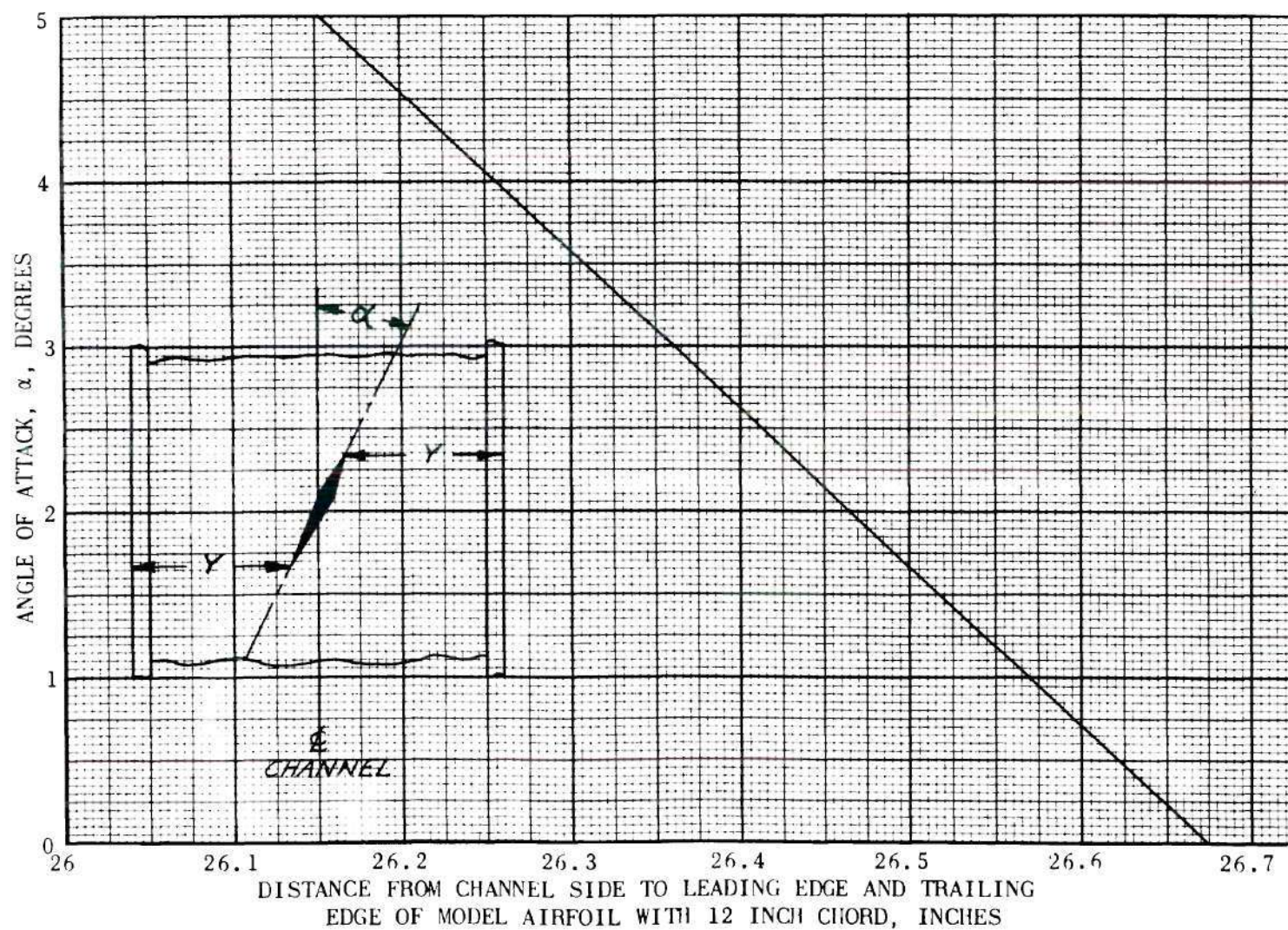
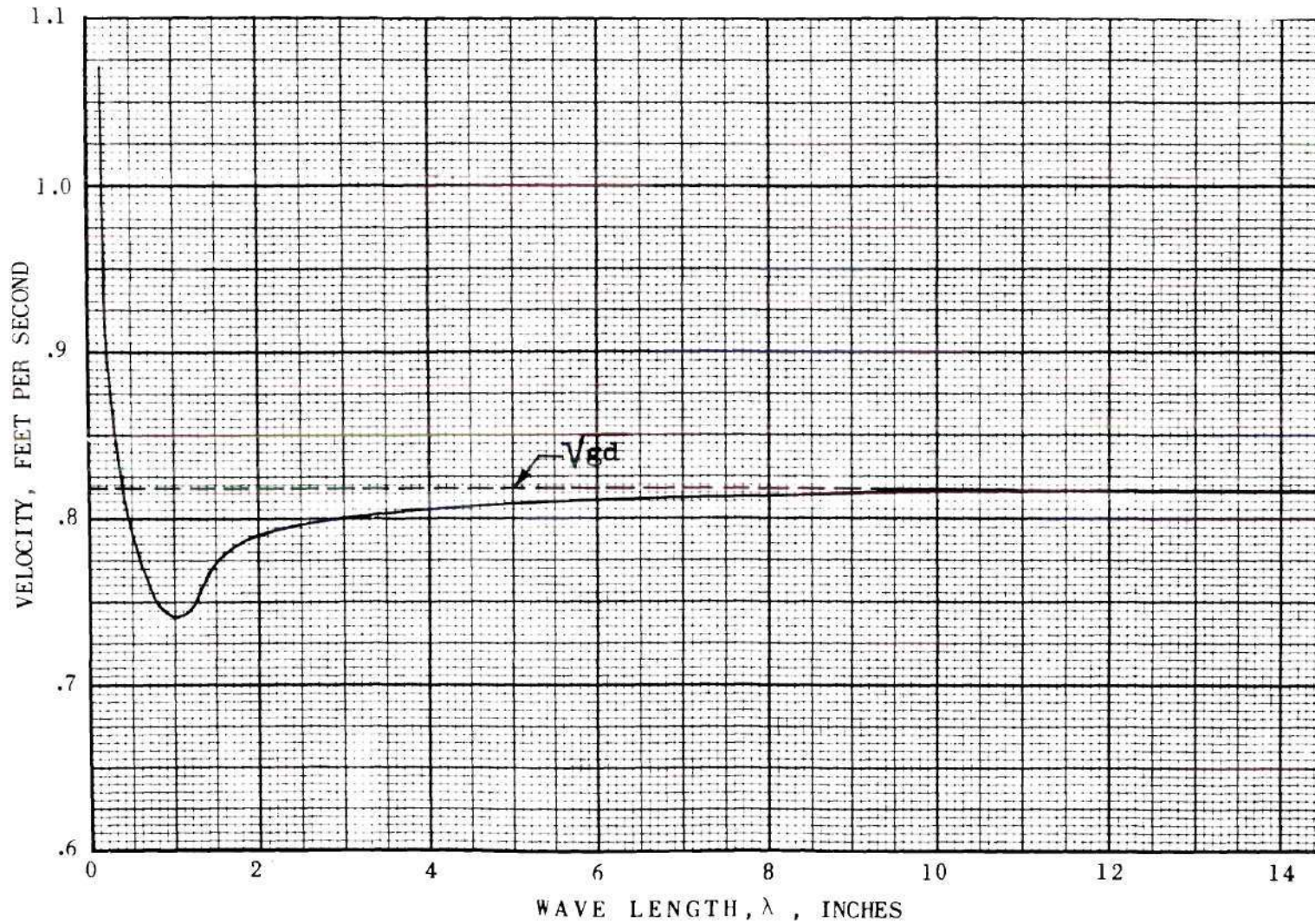
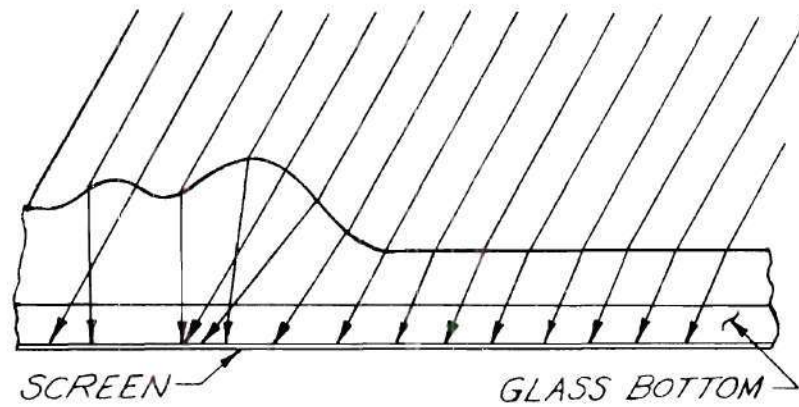


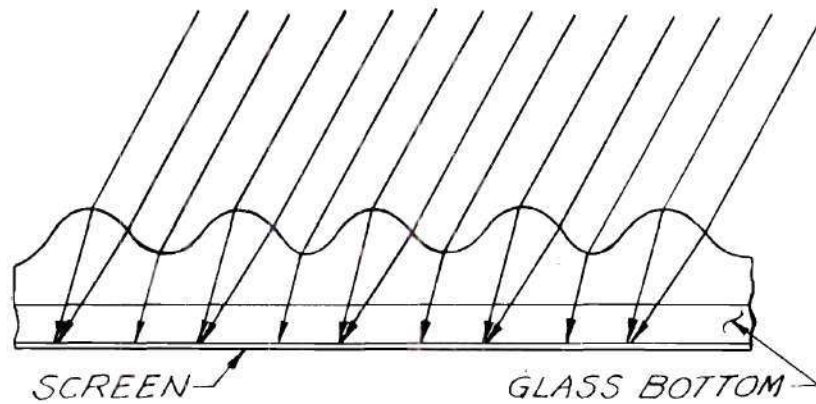


FIGURE 11  
VARIATION OF WAVE VELOCITY WITH WAVE LENGTH FOR  
A STATIC WATER DEPTH OF ONE QUARTER OF AN INCH



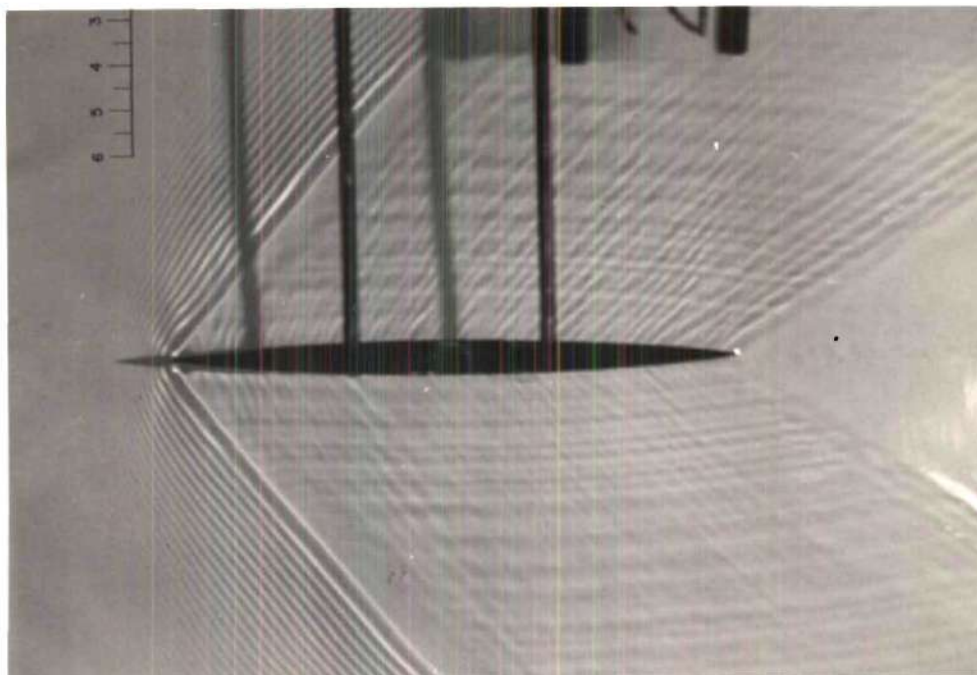


(a) UNDULAR HYDRAULIC JUMP



(b) CAPILLARY WAVES

FIGURE 12  
ANALYSIS OF REFRACTION PATTERNS FORMED BY AN  
UNDULAR HYDRAULIC JUMP, AND CAPILLARY WAVES



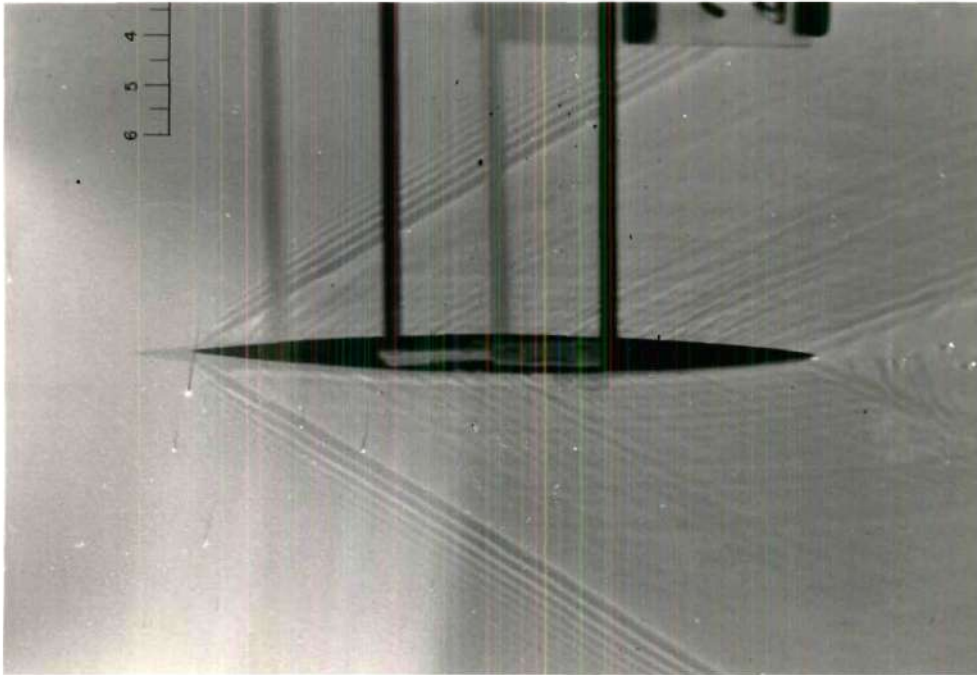
(a)



(b)

FIGURE 13  
FLOW ABOUT CIRCULAR ARC AIRFOIL,  $M=1.56$



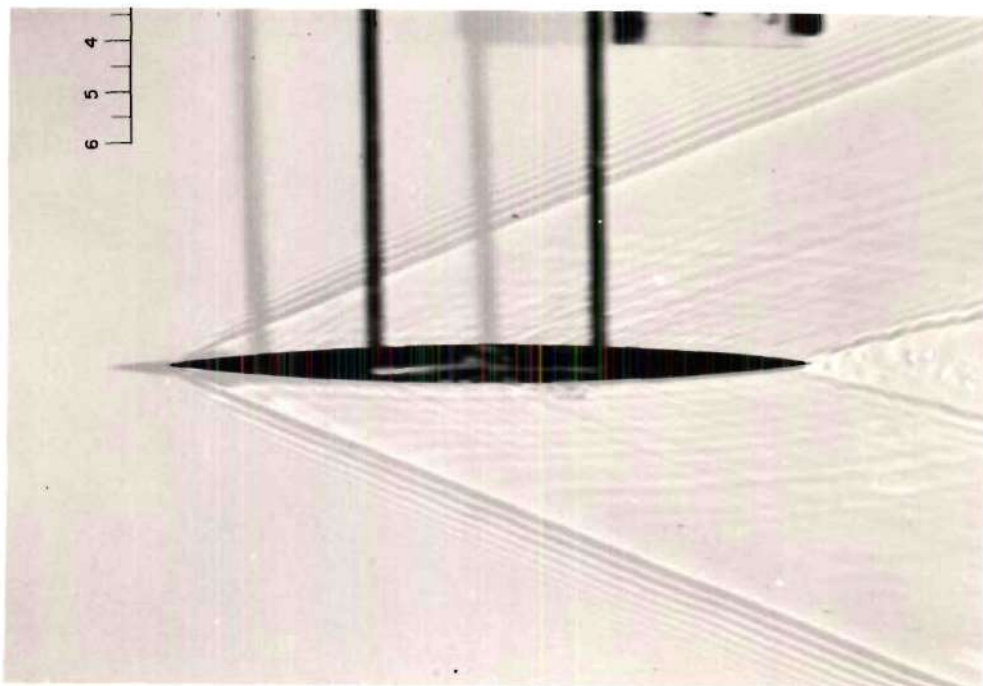


(a)

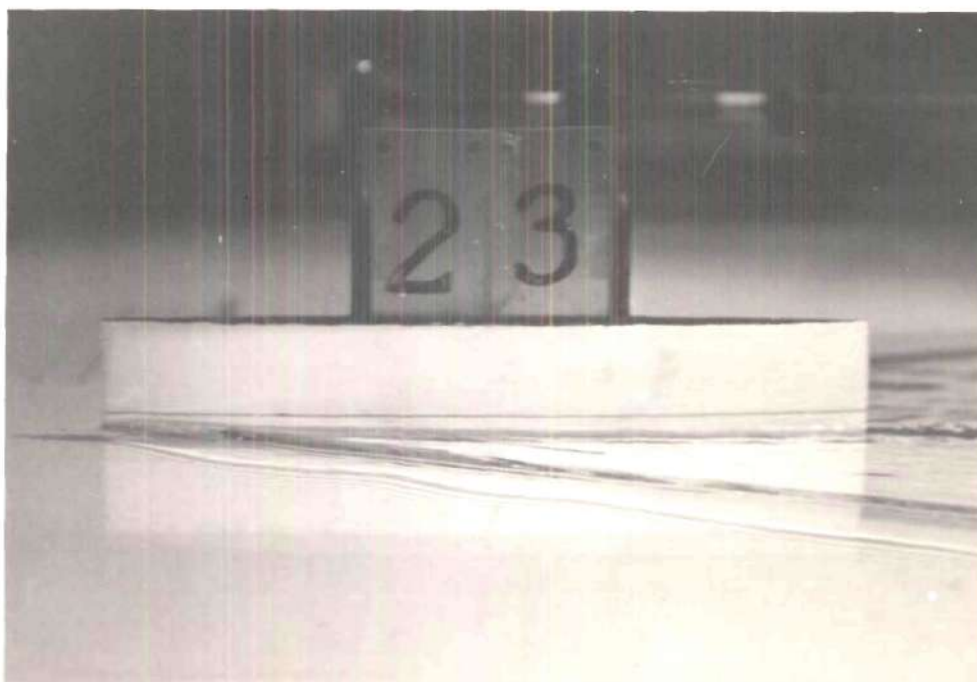


(b)

FIGURE 14  
FLOW ABOUT CIRCULAR ARC AIRFOIL,  $M=2.34$

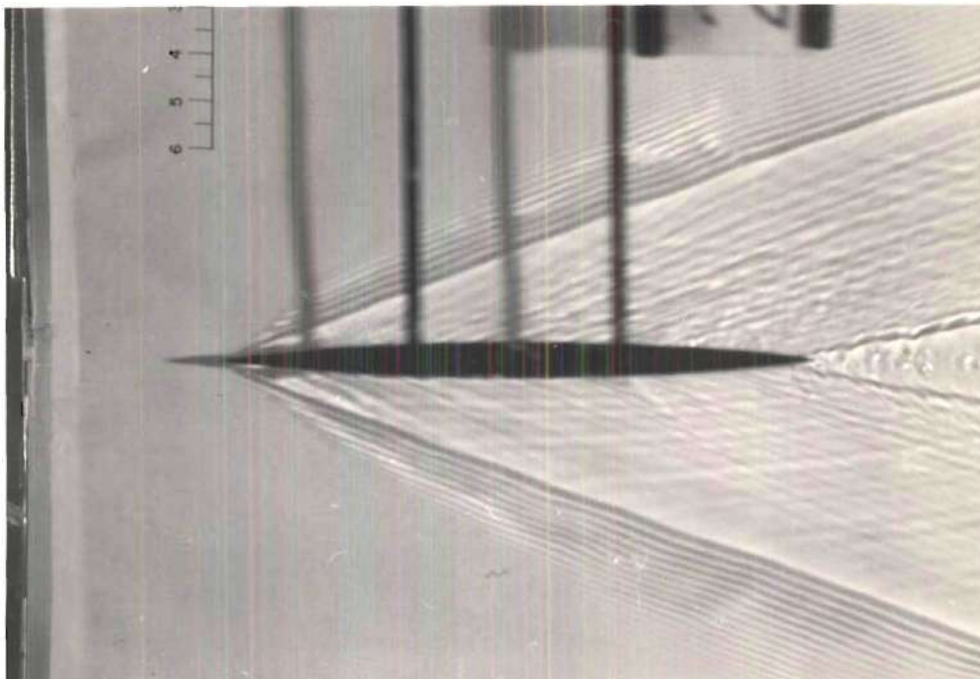


(a)

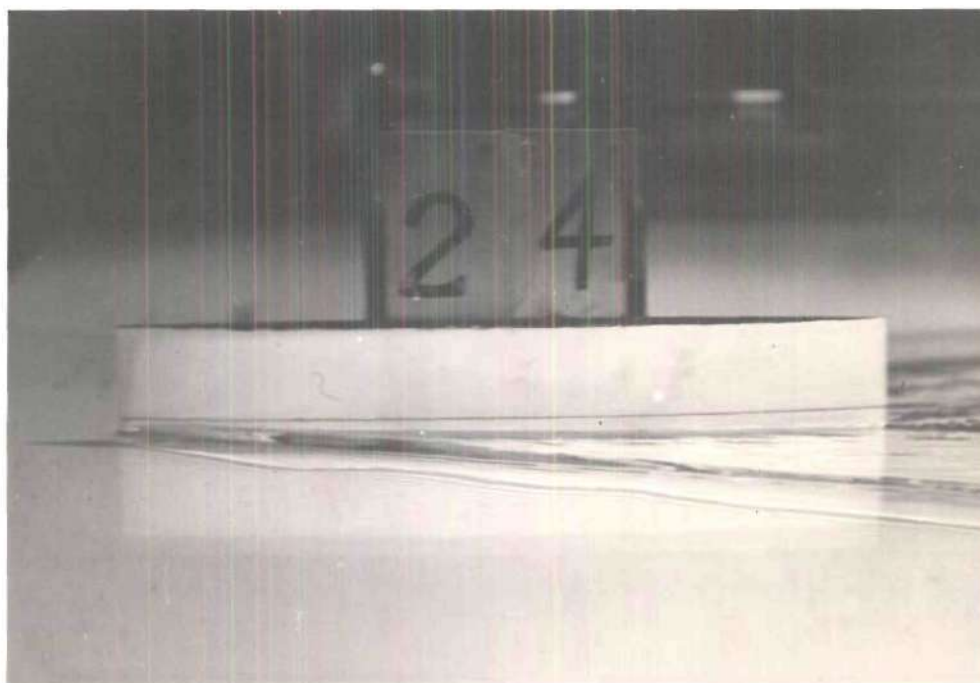


(b)

FIGURE 15  
FLOW ABOUT CIRCULAR ARC AIRFOIL,  $M=3.28$



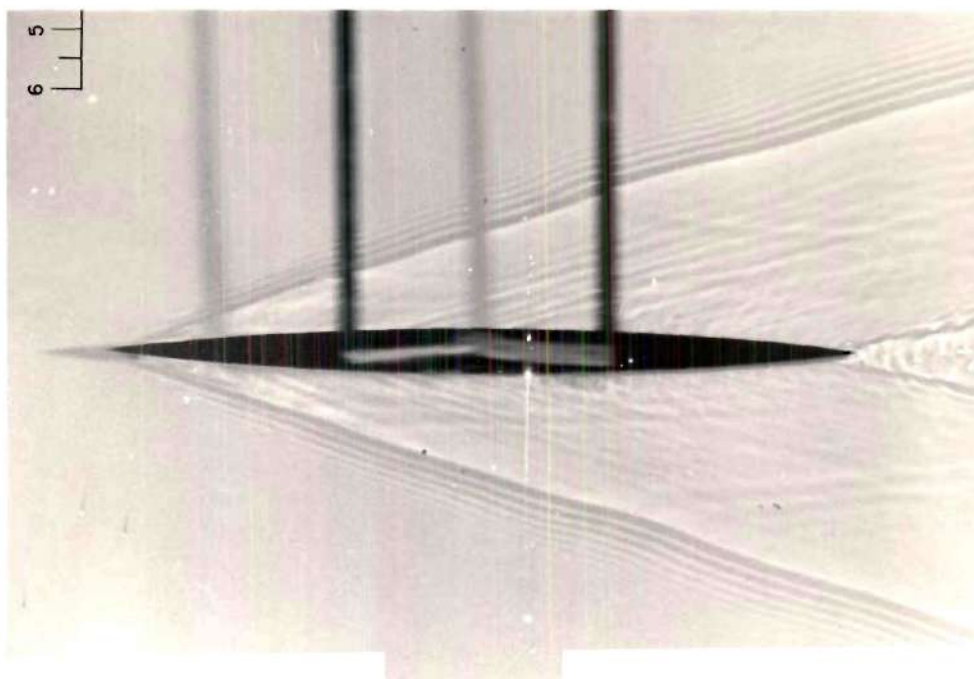
(a)



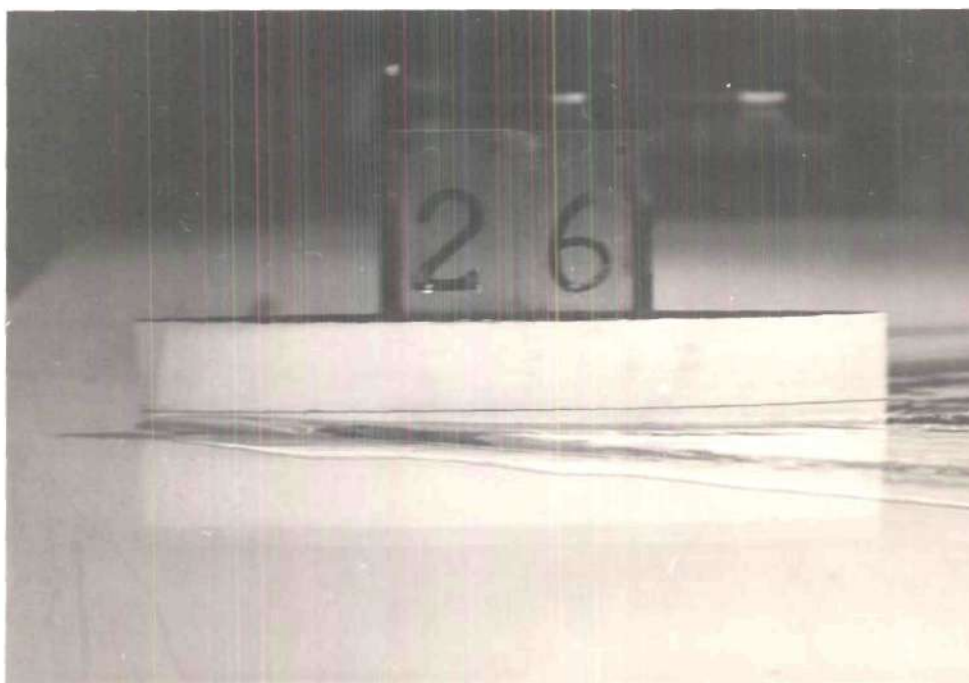
(b)

FIGURE 16  
FLOW ABOUT LENTICULAR AIRFOIL,  $M=3.71$



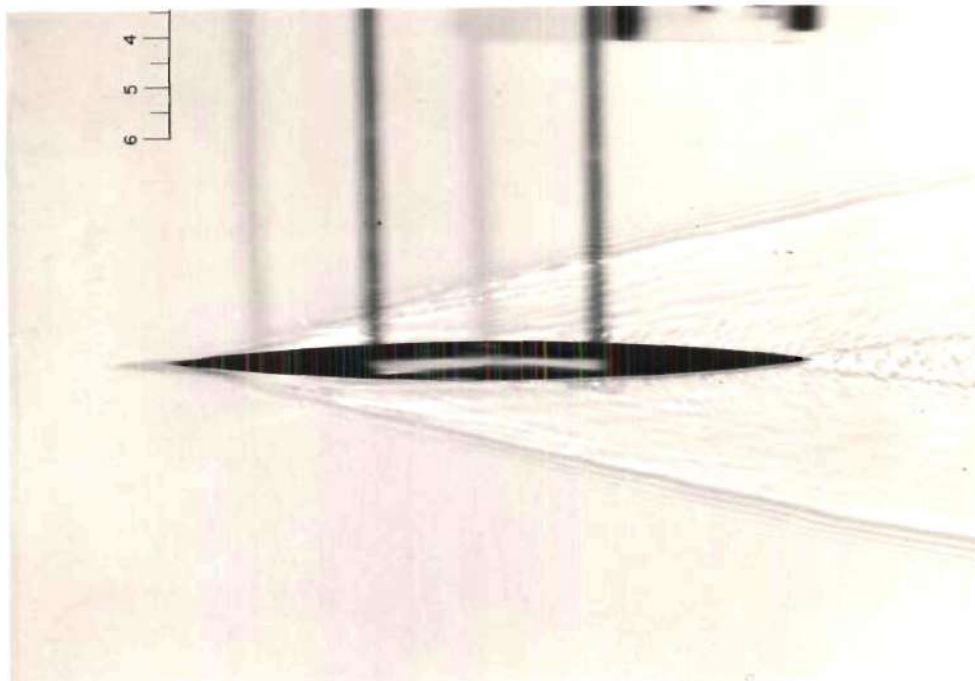


(a)



(b)

FIGURE 17  
FLOW ABOUT LENTICULAR AIRFOIL,  $M=4.26$

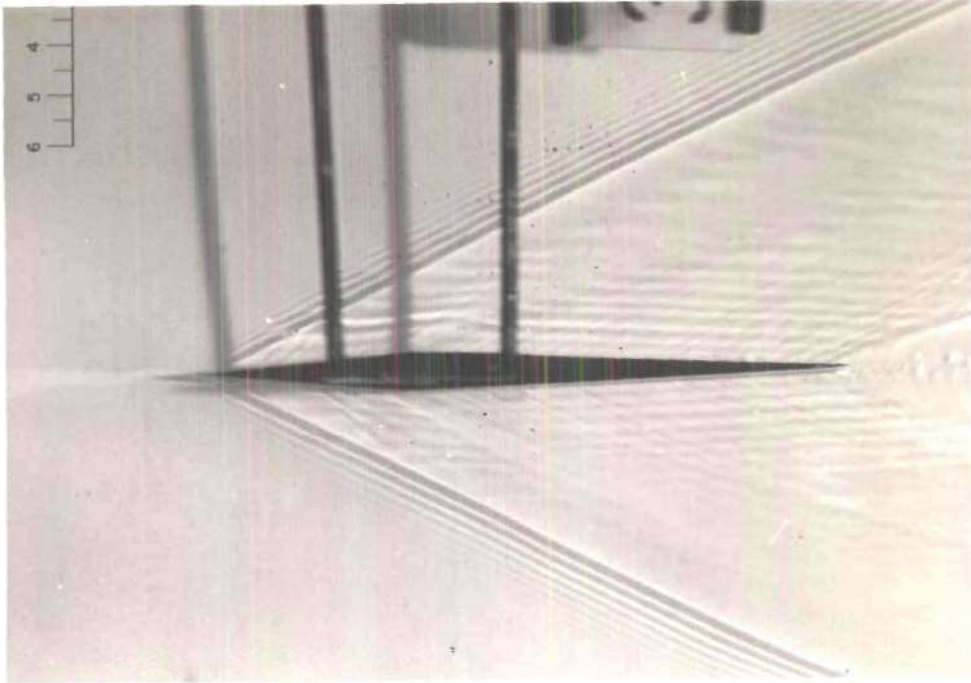


(a)



(b)

FIGURE 18  
FLOW ABOUT LENTICULAR AIRFOIL,  $M=6.78$



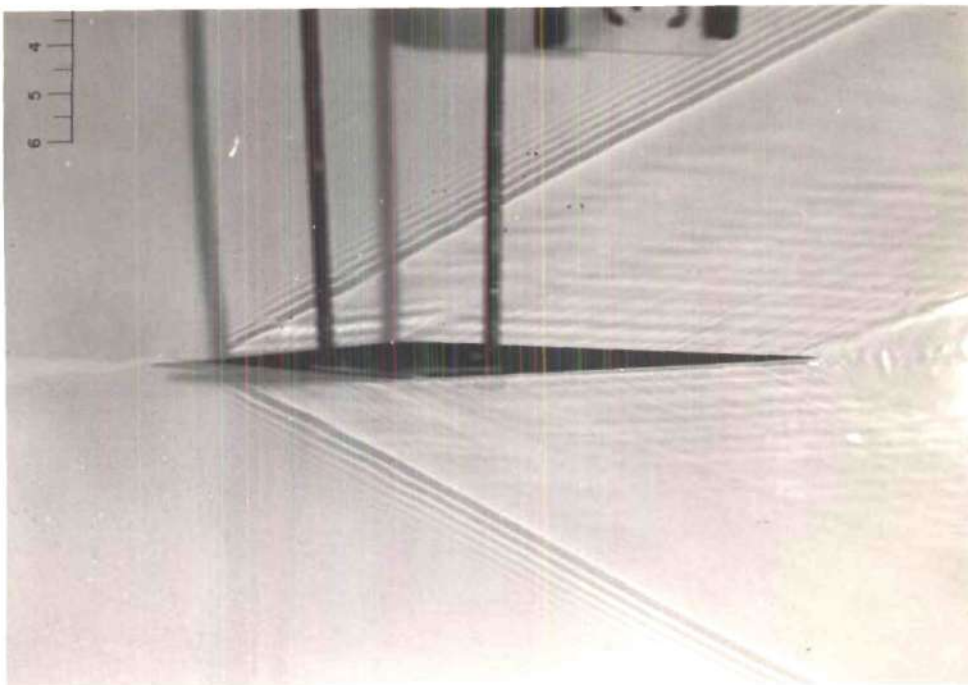
(a)



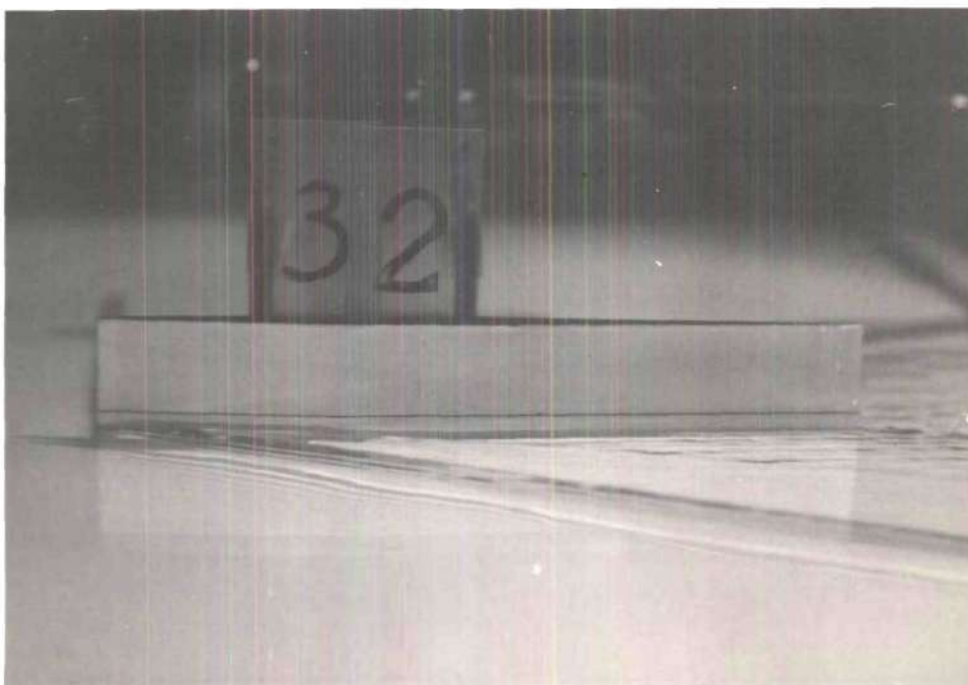
(b)

FIGURE 19  
FLOW ABOUT DIAMOND AIRFOIL,  $M=2.63$



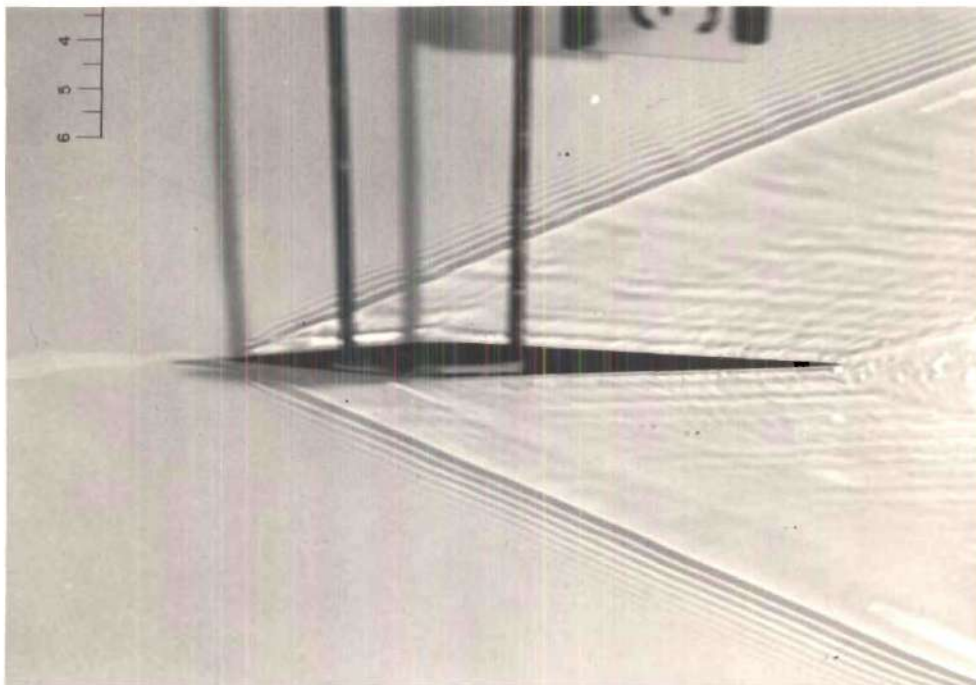


(a)

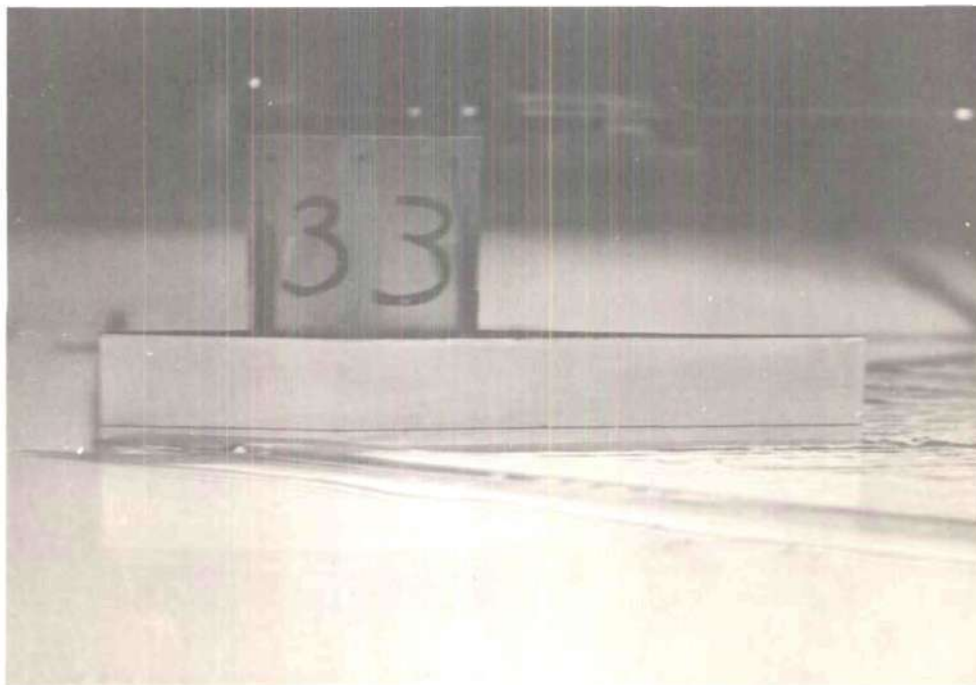


(b)

FIGURE 20  
FLOW ABOUT DIAMOND AIRFOIL,  $M=2.47$

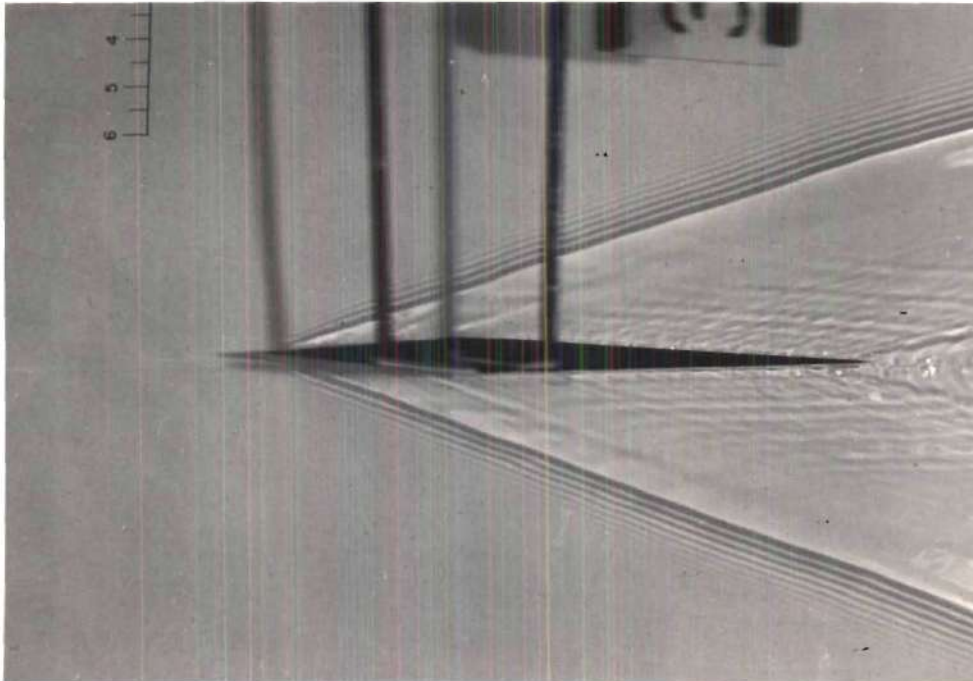


(a)



(b)

FIGURE 21  
FLOW ABOUT DIAMOND AIRFOIL,  $M=3.02$



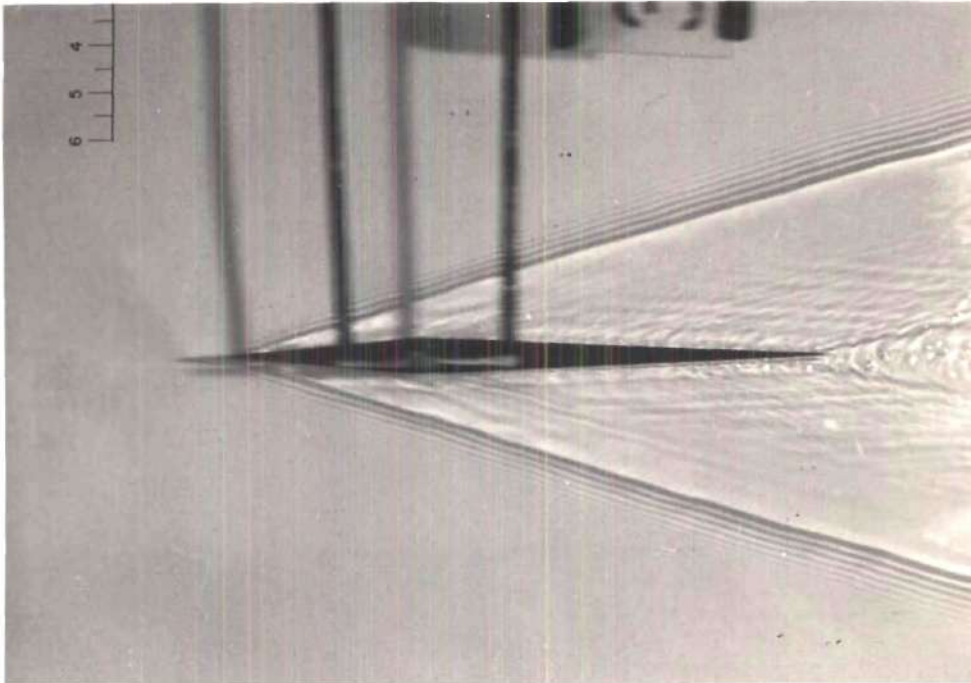
(a)



(b)

FIGURE 22  
FLOW ABOUT DIAMOND AIRFOIL,  $M=3.72$



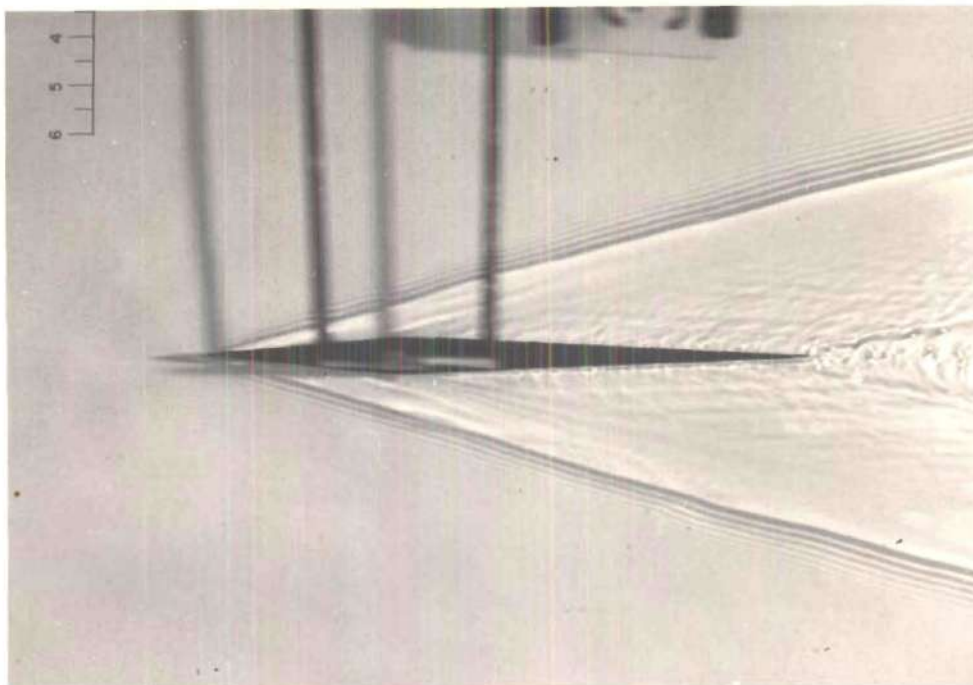


(a)



(b)

FIGURE 23  
FLOW ABOUT DIAMOND AIRFOIL,  $M=4.53$



(a)



(b)

FIGURE 24  
FLOW ABOUT DIAMOND AIRFOIL,  $M=5.08$

FIGURE 25  
VARIATION OF SHOCK WAVE ANGLE WITH FREE  
STREAM MACH NUMBER FOR DIAMOND AIRFOIL

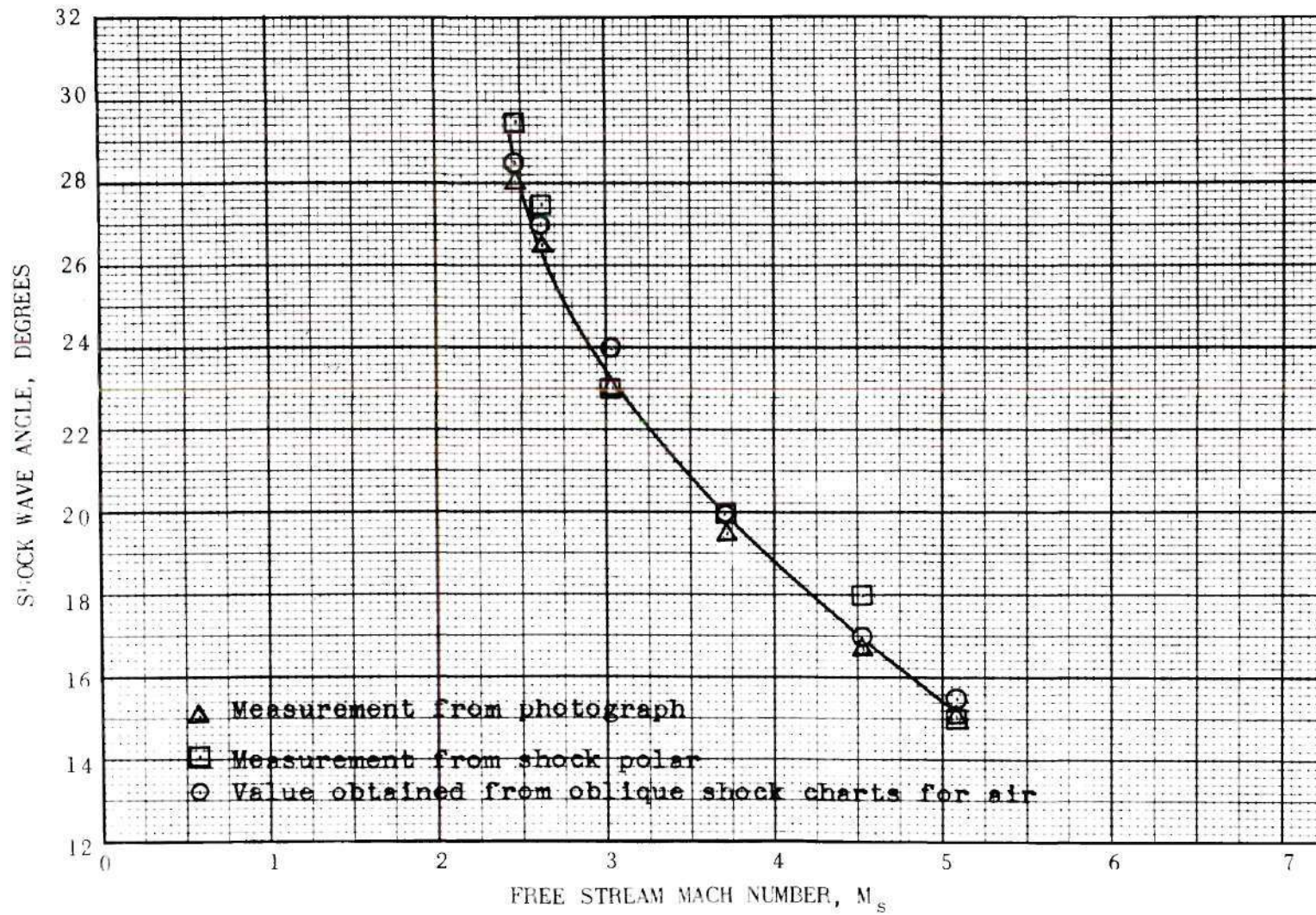




FIGURE 26

VARIATION OF SHOCK WAVE ANGLE WITH FREE STREAM  
MACH NUMBER FOR SYMMETRICAL CIRCULAR ARC AIRFOIL

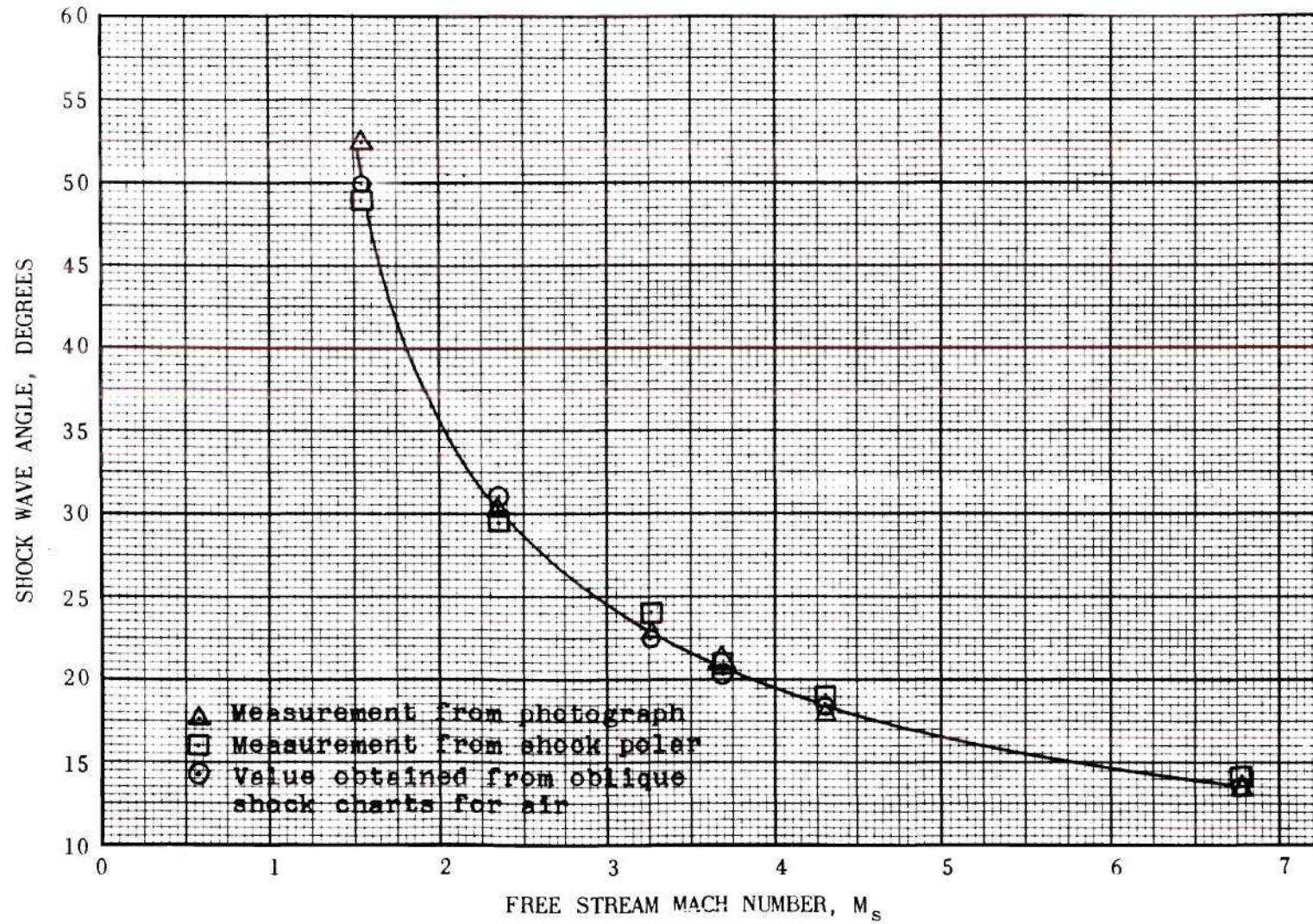




FIGURE 27

DIAMOND AIRFOIL LOCAL MACH NUMBER DISTRIBUTION  
FOR VARIOUS FREE STREAM MACH NUMBERS

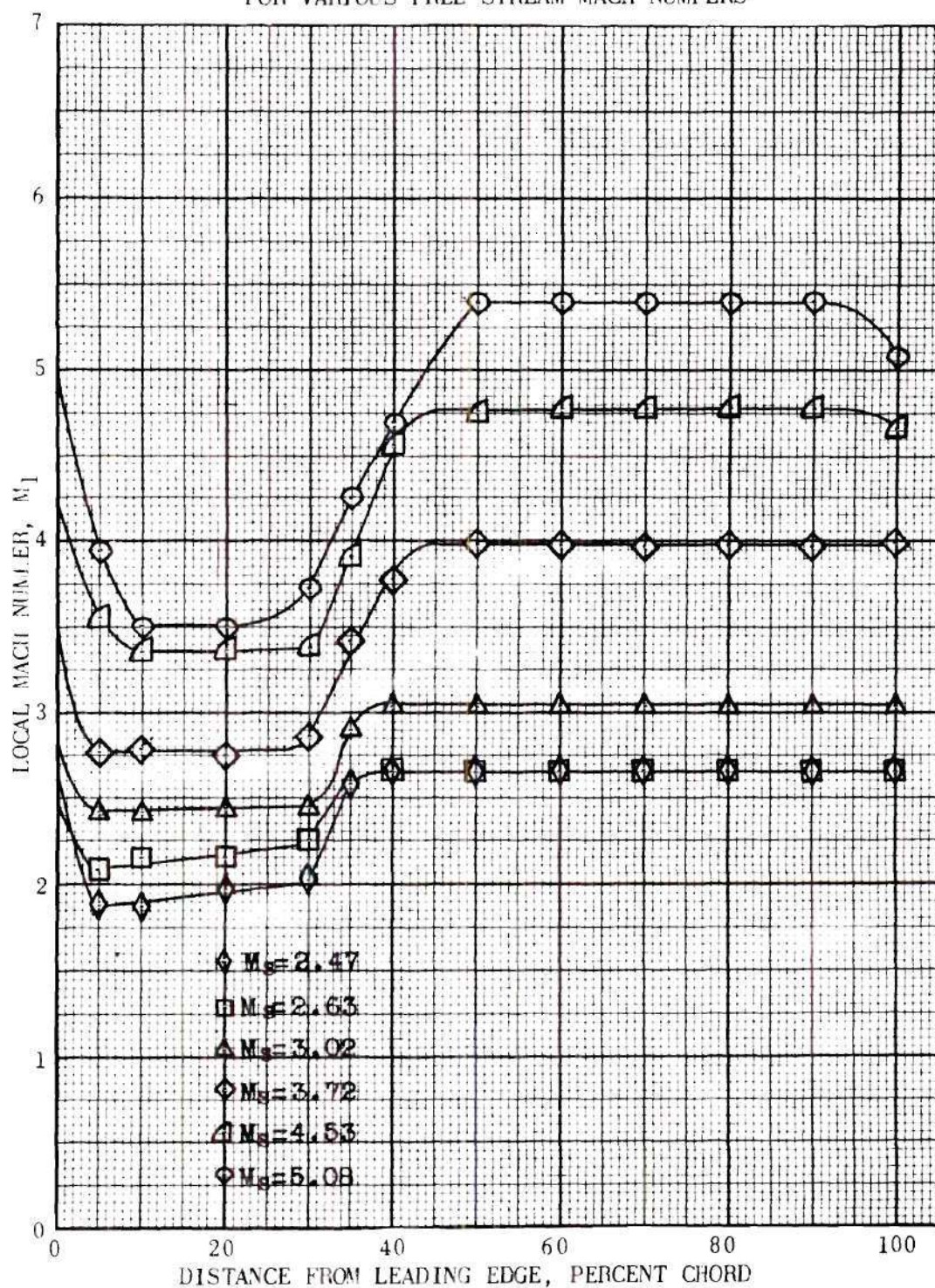




FIGURE 28

CIRCULAR ARC AIRFOIL LOCAL MACK NUMBER DISTRIBUTION  
FOR VARIOUS FREE STREAM MACH NUMBERS

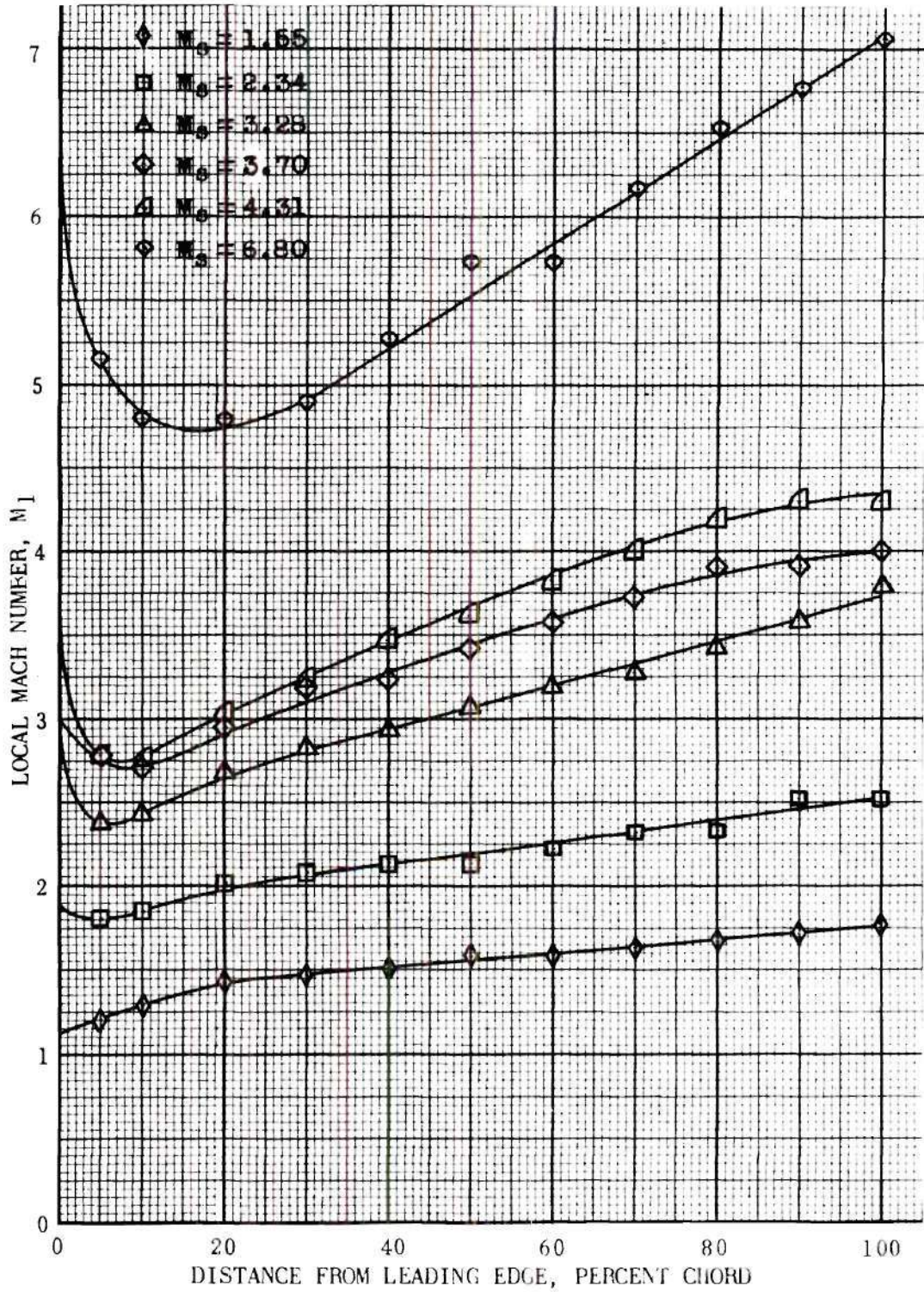




FIGURE 29  
PRESSURE COEFFICIENT FOR  
DIAMOND AIRFOIL AT  $M_s = 4.53$

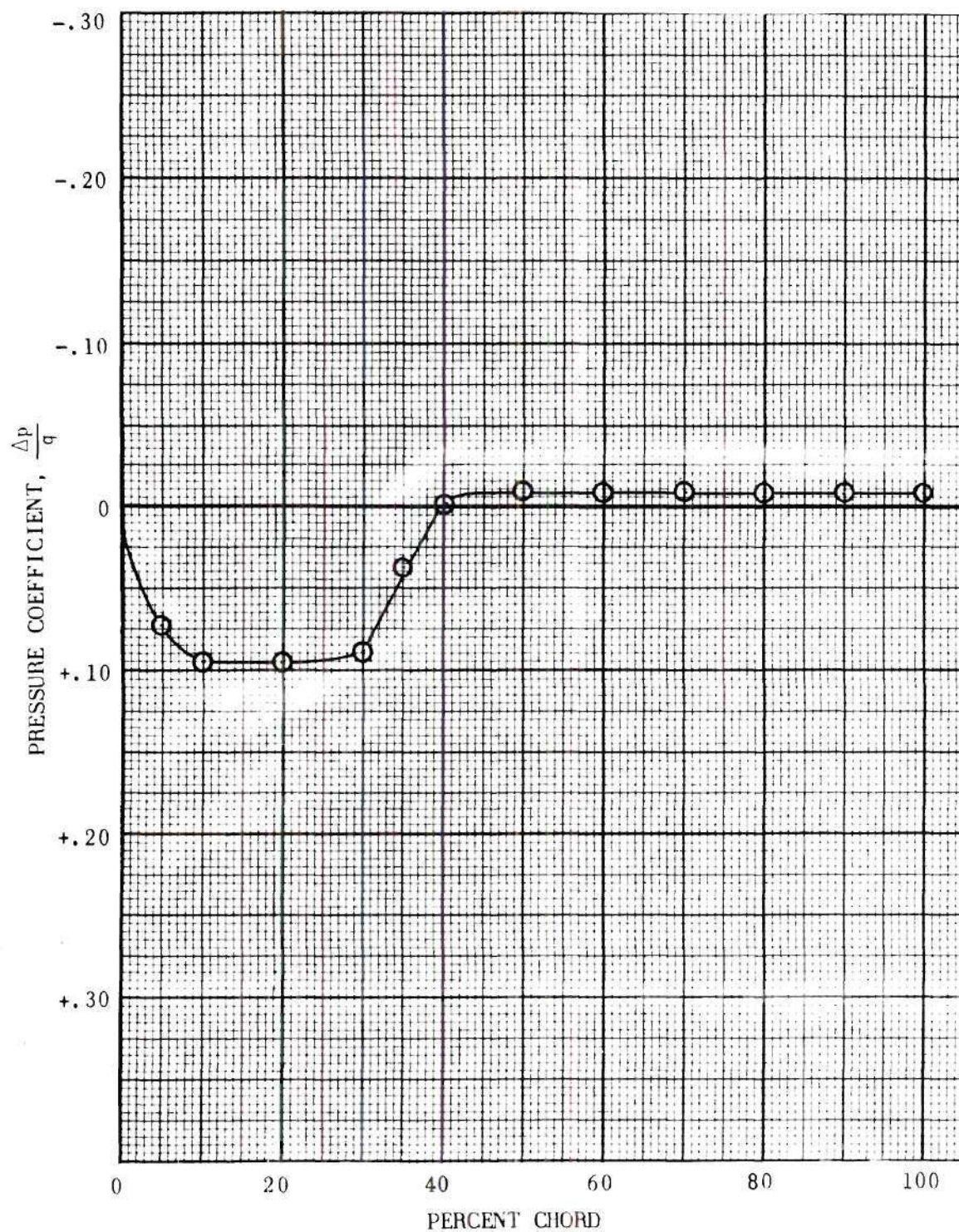




FIGURE 30  
PRESSURE COEFFICIENT FOR  
CIRCULAR ARC AIRFOIL AT  $M_s = 4.26$

